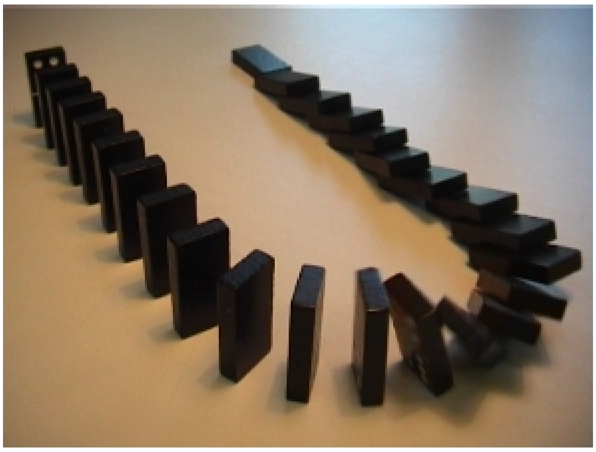
Robustness of Corroded Reinforced Concrete Structures



Short Term Scientific Mission, COST ACTION TU-0601

by

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under the guidance of

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and
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This report respects to the research done by the PhD Student, Eduardo Soares Cavaco from Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa, during a Short Term Scientific Mission (STSM) with regard to Cost Action TU-0601.

The STMS took place at Universitat Politècnica de Catalunya from 01/03/2009 to 31/07/2009 under the guidance of **P**rof. Juan R. Casas.

The propose of the visit was to intend to make contributions to the definition of structural robustness especially in the analysis of reinforced concrete structures subjected to corrosion.

An article on IABMAS 2010 is the expected publication resulting from the referred STSM.

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Abstract

The deterioration of existing structures has been a growing concern in the last decade. Significant attention has been paid to the deterioration of bridges, since significant costs in repair and replacement are expected in the next decades.

At the same time structural robustness seems to be an emergent concept related to the structural response to damage. At the present time, robustness is not well defined and much controversy still remains around this subject.

This report intends to be a contribution to the definition of structural robustness especially in the analysis of reinforced concrete structures subjected to corrosion.

To achieve this, first of all, several proposed robustness definitions and indicators and misunderstood concepts will be analyzed and compared. From this point and regarding a concept that could be applied to most type of structures and damage scenarios, a robustness definition is proposed.

To illustrate the proposed concept, some example of corroded reinforced concrete structures will be analyzed using nonlinear analysis numerical methods based on a continuum strong discontinuities approach and isotropic damage models for concrete.

Finally the robustness of the presented examples will be assessed and compared.

Chapter 1

Introduction

Tragic events such as the partial collapse of Ronan Point building (London, UK - 1968) or most recently the collapse of the World Trade Center (New York, USA - 2001) and the I-35W Mississippi River bridge (Minneapolis, USA - 2007), among others, increased the attention of engineers, but also the society, to the safety, reliability and robustness of structures.

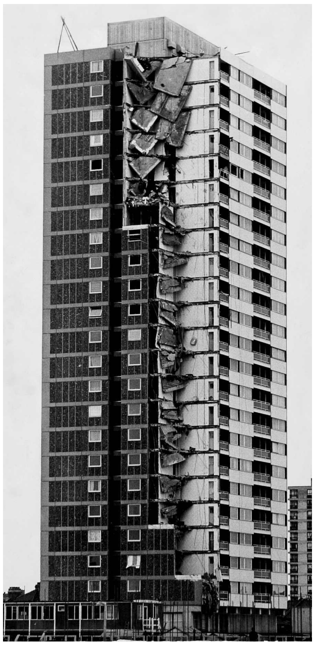

Figure 1.1: Ronan Point (London, UK - 1968).

Ronan Point was a 23-storey tower block in Newham, East London, which suffered a fatal partial collapse due to a natural gas explosion on a kitchen located on a corner flat on the 18th floor of the building (Figure 1.1). The Ronan Point tower consisted of precast panels joined together, without any structural frame, and thus lacked alternate load paths to redistribute forces in the event of a partial collapse. When the structure was dismantled, investigators found poor workmanship of the critical connections between the panels. The already shaky structure had been further weakened by the inadequate construction practices (Pearson et al., 2003). The structure was described by Levi and

Salvadori (1992) as a "*house of cards*".

Figure 1.2: September 11 (New York, USA - 2001).

The collapse of World Trade Center Towers on September 11, Figure 1.2, was triggered by an impact of two commercial aircrafts that damaged partially the perimeter steel tube design. In spite of the redundancy of the system the prolonged exposure to high temperatures lead to additional steel connection failures, resulting in failure of critical floor over the already weaken floor below. This started a chain reaction leading to the overall ruin of the towers (Eagar and Musso, 2001).

The initiating event in the collapse of the I-35W bridge (Figure 1.3) was the lateral shifting instability of the upper end of a diagonal member and the subsequent failure of a gusset plates (Figure 1.4) on the center portion of the deck truss. Since the deck truss portion of the I-35W bridge was non-load-path redundant, the total collapse of the deck truss was likely once the gusset plates failed (NTSB, 2008).

What seems to be common to these cases is the occurrence of consequences disproportionate to the initial cause or damage.

Society is demanding robust and reliable structures, but what society needs to take conscience is that no building system can be engineered and constructed to be absolutely risk-free in the presence of numerous sources of uncertainties that arise in the building process or from potential failure-initiating events. Building codes and standards just provide tools for structural engineers to manage risk in the public interest (Ellingwood and Dusenberry, 2005).

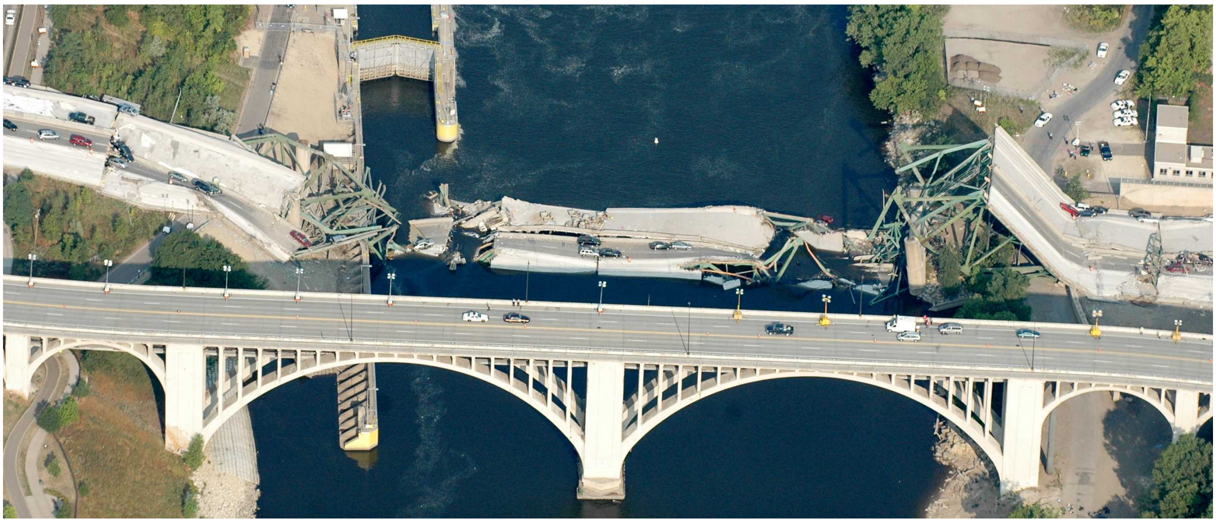

Figure 1.3: I-35W Mississippi River bridge collapse (Minneapolis, USA - 2007).

Most modern structural design codes provide detailed direction for verifying if a design is acceptable with regard to individual failure modes, which in most cases, relate to damage of individual structural components. Unfortunately the codes are far less specific, regarding requirements for reliability against system failure (Baker et al., 2008). Often, the stated requirement is that structural systems should be robust but a precise definition of structural robustness still does not exist.

This limitation is more important as most structural failures are due to unexpected loads, design errors, errors during execution, unforeseen deterioration and poor maintenance which can not be prevented using conventional component based code checking formats (Canisius et al., 2007).

Figure 1.4: Gusset plate used to unite multiple structural members of a truss (adapted from NTSB, 2008).

Regarding this scenario, the question arises of what can be done to improve this situation, since there are no risk free structures, actual design codes do not regulate against system failures and the main causes of structural collapse can not be predicted and avoided.

There are no simple answers for this question, but certainly the risk could be minimized if structures would be designed to be less vulnerable to local damage no matter what causes it, or in other words if structures would be more robust. To achieve this it is important to have a precise definition of robustness. Then it would be possible to calibrate it and introduce it on design codes. After this, comparing different design solutions and choosing between the more robust would be feasible.

This report intends to be a contribution to the definition of structural robustness. To do it, first of all, several proposed robustness definitions and indicators and misunderstood concepts will be analyzed and compared. From this point, and regarding a concept that could be applied to most type of structures and damage scenarios, a robustness definition is proposed.To illustrate the proposed concept, some examples of corroded reinforced concrete structures will be analyzed using methods of nonlinear analysis based on continuum strong discontinuities approach and isotropic damage models for concrete. Finally the robustness of the presented examples will be assessed and compared.

Chapter 2

Robustness Definition

2.1 Introduction

Several attempts to define robustness were made by numerous authors, but consensus has still not been reached. Here several proposed definitions for robustness are presented, including some originating from areas other than structural engineering.

Robustness can be defined as:

1. *"The consequences of structural failure are not disproportional to the effect causing the failure"* (CEN, 1994).

2. *"The ability of a structure to withstand extreme events without being damage to an extent disproportionate to the original cause"* (Agarwal et al., 2006).

3. *"...defined as insensitivity of a structure to local failure. It's a property of the structure alone and independent of the possible causes and probabilities of the initial local failure"* (Starossek, 2008).

4. *"...bridge robustness, ability to carry loads after the failure of one of its members"* (Wisniewski et al., 2006).

5. *"The ability to react appropriately to abnormal circumstances (i.e. circumstances "outside of specifications"). A system may be correct without being robust"*(Meyer, 1997).

6. *"The ability of a system to maintain function even with changes in internal structure or external environment"* (Callaway et al., 2000).

7. *"...robustness is taken to imply tolerance to damage from extreme loads or accidental loads, human error and deterioration"*(Baker et al., 2008).

8. *"We call a system robust if it can withstand an arbitrary damage, for example, the loss of a member or degradation in the quality of a member"* (Agarwal et al., 2006).

9. *"The degree to which a system is insensitive to effects that are not considered in the design"* (Slotine et al., 1991).

10. *"Insensitivity against small deviations in the assumptions"* (Huber, 1996).

As was seen above, several concepts were used to define structural robustness. The fundamental concepts are: event, causes, damage, environment, function, consequences. Disproportionate or abnormal were also used but not on a defining sense but instead on a quantifying one.

From the definitions presented, it can be concluded that robustness is a property relating causes, events and damage with consequences and structural functions. If the relation is proportionate the structure is robust, if not, the structure is not robust (Figure 2.1).

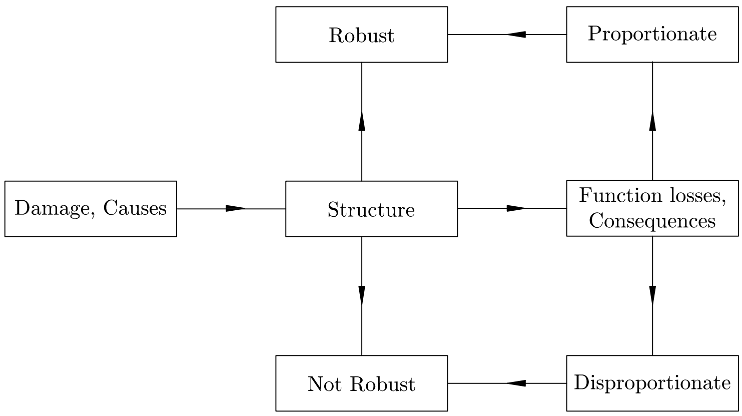

Figure 2.1: Defining Robustness.

Now, let's see how the referred concepts are related and how they are different from each other. Given a structure built on a specified environment subjected to an event, some damage may arise depending on type and magnitude of the event. The damage could lead to some local or system failure or, in more general words, to some degree of loss of structural function. Depending on the damage, consequences may arise to both structure and environment and a new cycle may begin leading to progressive loss in structural function and in some cases to full collapse (Figure 2.2).

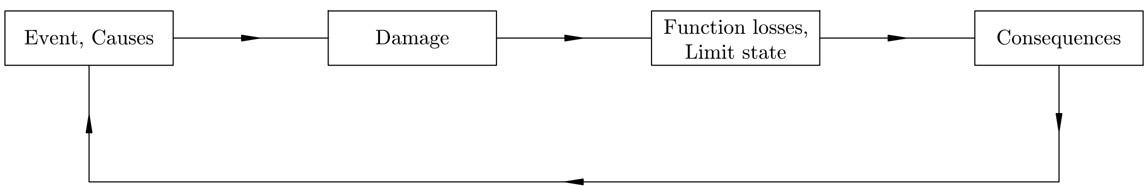

Figure 2.2: Progressive loss in structural function.

Let's analyze the above definitions and relate them with the initial given examples of structural collapses where the lack of robustness was recognized.

On the first and second definitions, and for the Ronan Point example, it can be accepted that the consequences were large, but it's difficult to accept that the cause of damage or the trigger event, a gas explosion, was trivial. In fact a gas explosion is an extreme event. What can be accepted is that the initial damage itself, and not the cause of it, when compared with the overall damaged was insignificant.

According with Starossek (2008), robustness is also related with the initial damage, and not with the "possible causes and probabilities of the initial local failure", being a property of the structure.

Although the first two definitions seem similar, they are slightly different. In the second one, after the trigger event happens, it's referred the structure ability to maintain function and on the first one, the amount of consequences. The second definition looks just for the structural response and the first looks also for the environment, if consequences are interpreted in a broad sense.

In fact, from these definitions, it results that robustness can be defined as a property of both environmental and structure (Figure 2.3).

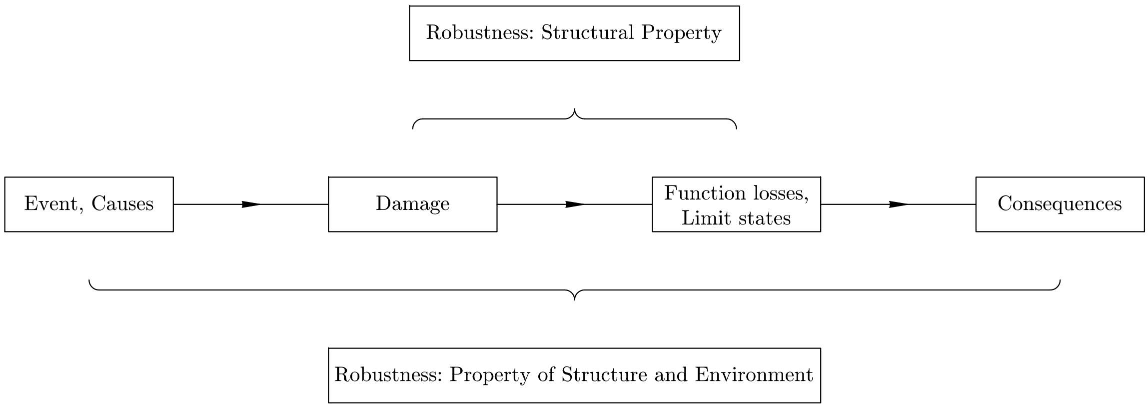

Figure 2.3: Robustness: structural property *vs.* property of structure and environment.

If robustness is considered a property of the structure, neither the causes of damages nor the environmental consequences of the damage can be included in the analysis and only the structure response and the initial damage should be considered. Definition 4 (Wisniewski et al., 2006) focus on this aspect, but limits the damage to a member failure. The point of view adopted in this report is that robustness can be regarded even when the damage is something trivial as corroded reinforcement or in much worst situation as damages resulting from a terrorist attack. Damage would be the same as defined by Yao (1985) and adopted by Frangopol and Curley (1987), i.e., damage refers to any strength deficiency introduced during design or construction phase of the structure as well as any deterioration of strength caused by external loading and/or environmental conditions during the lifetime of the structure. Thus a constructed structure can be considered to have an initial damage even before it has been subjected to any environmental loadings. In general then, damage can exist in the initial structure or be imposed upon the structure progressively or suddenly.

It's also a point of view that when talking about maintaining structural function after damage, it can be referred to the preservation of all kind of functions a structure is designed for. Definitions number 5, 6, 7, 8, 9 and 10 go on this direction, by not reducing damage and functions spectrums.

As a conclusion, and if considering only a structural property it can be said that robustness is a measure of the degree of structural function loss after damage occurrence. This relation can have many forms, from service limit states to ultimate limit states.

Damage can vary from a simple degradation state to a more serious damage as a column or a beam failure. Errors during the design or the construction stage can also be seen as types of damages. The concept behind this definition is to limit neither the functions spectrum nor the damage scenarios.

2.2 Related Concepts

"Continuous, highly redundant structures with ductility tend to absorb local damage well. Other systems, such as large panel or bearing wall systems, pre-cast concrete slabs or steel joint floors supported on masonry walls, and any building system that is well tied but lacks ductility are inherently more vulnerable because of the difficulties in providing continuity and ductility in such systems" (Ellingwood and Dusenberry, 2005).

As can be seen the above citation includes important robustness related concepts sometimes used with the same meaning. Here some robustness related concepts definitions will be proposed without the pretension to give the true ones, but instead the point of view adopted in this report:

1. Vulnerability - It refers to the damage susceptibility of a structure to environmental exposure. It's not a structural property. The same structure can be more or less vulnerable depending on the overall situation. For example the same bridge can be vulnerable to corrosion if it's located on a maritime environmental but not vulnerable if it's located on chloride free environment. Another example, *"a wooden house is less likely to collapse in an earthquake but it may be more vulnerable in the event of a fire"* (Agarwal et al., 2006).

2. Damage Tolerance - Can be viewed as robustness synonymous. It's the ability to maintain structural performances after damage occurrence.

3. Progressive Collapse - On some structures damage on a component will overload another component that will become also damage initiating an overloading/damage chain reaction that leads to structure collapse. Progressive collapse is difficult to predict since it is associated with non-linear and dynamic behavior of the structure. If structural behavior under damage is hard to predict, the dynamic behavior under damage it's even harder. Robust design will minimize the risk of progressive collapse because the initiation of the chain reaction of overloading and damage is avoided.

4. Static Indeterminacy - It's the number of restrains and constrains above the minimum absolutely necessary to achieve an equilibrium configuration under any kind of load arrangement. Static Indeterminacy is a structural property.

5. Redundancy - As defined by Frangopol and Curley (1987), redundancy refers to the multiple availability of load-carrying paths a structure contains. As it depends on the load is not the same as static indeterminacy but is intently related to it. A structure can provide several load-carrying paths for one kind of load arrangement and few, or even none, for another different load configuration. The ability to provide a specific load-carrying path may depend on the competence to deform.

6. Flexibility - It's related with the structure ability to suffer elastic and reversible deformations.

7. Ductility - It's related with the structure ability to suffer plastic deformations with energy release.

8. Reliability - Refers to the probability of not exceeding limit state functions. On the other hand, robustness, as used in this report, is similar but is related with not exceeding limit state functions under a damage condition.

Chapter 3

Robustness Assessment

Since robustness is a desirable structural property for structural systems, it is paramount to be able to assess it, in order to compare between different design solutions and to choose the more adequate in a given context. If robustness could be assessed it would be possible to optimize it during design process.

In the following sections, several authors attempts to assess robustness or other related concepts are presented.

3.1 Frangopol and Curley, 1987

Frangopol and Curley (1987) analyzed the effects of damage and redundancy on structural systems proposing both deterministic and probabilistic measures for the latter.

On the deterministic approach the measure of redundancy is the reserve strength between components damage and system collapse, and can be defined by the following expression:

$$R = \frac{L_{Intact}}{L_{Intact} - L_{damaged}} \tag{3.1}$$

where L_{Intact} is the overall collapse load of the structure without damage and can be computed through plastic methods of structural analysis. $L_{damaged}$ is the overall collapse load of the structure considering some damage in one or more members. The redundancy factor is equal to 1 when the damaged structure has no reserve strength and is infinite when the damage has no influence on the reserve strength of the bridge.

To account for the random nature in safety evaluation of damaged structures, Frangopol and Curley (1987) also propose a probabilistic redundancy factor β_R defined by:

$$\beta_R = \frac{\beta_{Intact}}{\beta_{Intact} - \beta_{damaged}} \tag{3.2}$$

where β_{Intact} is the reliability index of the intact system and $\beta_{damaged}$ is the reliability index of the damaged system. Similarly, if probabilistic redundant index takes values close to infinite then structure is very robust. If probabilistic redundant index assumes values close to 1 that means robustness is null.

3.2 Lind, 1995

Lind (1995) proposes quantitative measures of vulnerability and damage tolerance of a system. In his point of view, vulnerability and damage tolerance are complementary concepts. If a system is vulnerable it is not damage tolerant and vice versa.

As can be seen vulnerability here has a different sense than the adopted on the previous chapter. In this report vulnerability was defined as the susceptibility to environmental exposure and Lind defines it as the damage tolerance, what was defined as robustness.

The vulnerability V of a system is defined as:

$$V = \frac{P(r_0, S)}{P(r_d, S)} \tag{3.3}$$

where r_d is the resistance of the damaged system, r_0 is the resistance of the intact system, and S is the loading. $P(r, S)$ is the probability of system failure as a function of both effect of loading and resistance. The vulnerability V of a system can vary from zero to infinite, if the damage has null or huge impact on system resistance, respectivelly.

On the other hand damage tolerance T_d can be viewed as the inverse of vulnerability V:

$$T_d = \frac{P(r_d, S)}{P(r_0, S)} \tag{3.4}$$

As explained later, Lind's indicator to assess robustness is very similar to the one suggested by Frangopol and Curley (1987). They represent a form to measure robustness as a property of the structure with the advantage that can be applied to any kind of damage and structural performance.

3.3 Ghosn and Moses, 1998

Ghosn and Moses (1998) focused on bridges, defining redundancy as the ability of the structure to continue to carry load after the failure of one of its members. Redundancy is defined as described on Chapter 2 if damage corresponds to the member failure and function matches to load carrying capacity.

As a matter a fact what Ghosn and Moses (1998) proposed is an entire methodology to assess, not just the member, but all system safety. It is assumed that a bridge may be considered safe from a system view point if:

- it provides a reasonable safety level against first member failure;
- it does not produce large deformations under regular traffic conditions;
- it does not reach its ultimate system capacity under extreme loading conditions;
- it is able to carry some traffic loads after damage or the loss of a main load-carrying member.

Therefore, the following states should be checked to insure adequate bridge redundancy and system safety:

1. Member failure limit state: this is the traditional check of individual member safety and the corresponding level of safety may be represented by the reliability index β_{member}.
2. Serviceability limit state: this is defined as a maximum live load displacement accounting for the nonlinear behavior of the bridge system which correspond the value of β_{serv}.
3. Ultimate limit state: this is the ultimate capacity of the bridge system against the formation of a collapse mechanism which correspond the value of β_{ult}.
4. Damaged condition limit state: this is defined as the ultimate capacity of the bridge system after the complete removal of one main load carrying component from the structural model. The value of $\beta_{damaged}$ is defined in this situation.

The incorporation of system behavior to the safety assessment in the mentioned method is done by the relative reliability indices $\Delta\beta_i$, which are defined as the difference between the safety indices for the system and the safety index for the member. In order to guarantee the bridge safety, the obtained relative reliability indices must be greater than the corresponding target values and, at the same time, the member safety has to be ensured. The safety format should take the form above:

$$\beta_{ult} = \Delta\beta_{ult} + \beta_{member} \geq \Delta\beta_{ulttarget} + \beta_{membertarget} = \beta_{ulttarget} \tag{3.5}$$

$$\beta_{serv} = \Delta\beta_{serv} + \beta_{member} \geq \Delta\beta_{servtarget} + \beta_{membertarget} = \beta_{servtarget} \tag{3.6}$$

$$\beta_{damaged} = \Delta\beta_{damaged} + \beta_{member} \geq \Delta\beta_{damagedtarget} + \beta_{membertarget} = \beta_{damagedtarget} \tag{3.7}$$

Ghosn and Moses (1998) go further by proposing target system values for the referred reliability indices for highway bridges. For bridges superstructures the proposes targets are:

$$\Delta\beta_{ult} = \beta_{ult} - \beta_{member} \geq 0.85 \tag{3.8}$$

$$\Delta\beta_{serv} = \beta_{serv} - \beta_{member} \geq 0.25 \tag{3.9}$$

$$\Delta\beta_{damaged} = \beta_{damaged} - \beta_{member} \geq -2.70 \tag{3.10}$$

Later, Liu et al. (2001) recommended for highway bridges substructures the following targets:

$$\Delta\beta_{ult} = \beta_{ult} - \beta_{member} \geq 0.50 \tag{3.11}$$

$$\Delta\beta_{serv} = \beta_{serv} - \beta_{member} \geq 0.50 \tag{3.12}$$

$$\Delta\beta_{damaged} = \beta_{damaged} - \beta_{member} \geq -2.00 \tag{3.13}$$

3.4 Baker et al., 2008

Baker et al. (2008) proposed a risk-based interpretation for robustness. Robustness is assessed by computing both direct risk, which is associated with the direct consequences of potential damages to the system, and indirect risk, which corresponds to the increased risk of a damaged system. Indirect risk can be interpreted as risk from consequences disproportionate to the cause of damage, and so robustness of a system is indicated by the contribution of these indirect risks to total risk. The robustness index I_{Rob} is then defined as:

$$I_{Rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \tag{3.14}$$

and measures the ratio between direct risk and total risk. The index may assume values between zero and one. If the system is completely robust I_{Rob} is equal to one, if all risk is due to indirect consequences, then I_{Rob} is equal to zero.

To assess both direct and indirect risk, decision analysis theory and event tree formulation can be used (Figure 3.1):

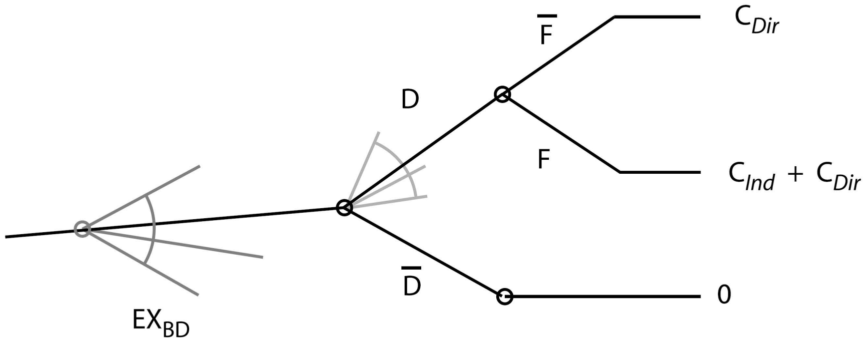

Figure 3.1: Event Tree (adapted from Baker et al., 2008).

First, an exposure EX_{BD} occurs which has the potential of damaging components in the system. If no damage $\bar{D}$ occurs, then the analysis is finished. If damage occurs, a variety of damage D states can result. For each of these states, there is a probability that system failure results F. Consequences are associated with each of the possible damage and failure scenarios.

To assess system risk the consequences associated to each scenario are multiplied by its occurrence probability, and then integrated over all the event space in the event tree. The risk corresponding to each branch is then:

$$R_{Dir} = \int_x \int_y C_{Dir} f_{D|EX_{BD}}(y|x) f_{EX_{BD}}(x) dy dx \tag{3.15}$$

$$R_{Ind} = \int_x \int_y C_{Ind} P(F|D = y) f_{D|EX_{BD}}(y|x) f_{EX_{BD}}(x) dy dx \tag{3.16}$$

where R_{Dir} and R_{Ind} are respectively the risk associated with direct (C_{Dir}) and indirect (C_{Ind}) consequences, $f_{D|EX_{BD}}$ is the damage subjected to a given exposure probability density function, $f_{EX_{BD}}$ is the exposure probability density function and $P(F|D = y)$ is the failure probability given a certain damage.

An exposure is considered to be any event that may cause potential damage to the system, from design loads, to accidental loads such as explosion or terrorist attacks or something more trivial such as exposure to agents that can cause deterioration. Damage refers to any performance reduction of structural components such as member failure, excessive deformation or material deterioration, among others. Likewise system failure may correspond to effective failure or just to a decrease on system performance.

As can be seen, the robustness definition proposed by Baker et al depends on environment and so cannot be considered only as a structural property. The same structure may have different robustness in different contexts. In addition the robustness of a structure may vary depending on changes (socio-economical) on the environment. So, over time, a structure that was robust may become not robust.

3.5 Biondini and Restelli, 2008

According to Biondini and Restelli (2008), robustness evaluations are usually related to damage suddenly provoked by accidental actions, like explosion or impacts. However, damage could also arise slowly in time from aging of structures, as induced, for example, by environmental aggressive agents. In this context, it is of great interest to develop suitable life-cycle measures of structural robustness with respect to a progressive deterioration of the structural performance.

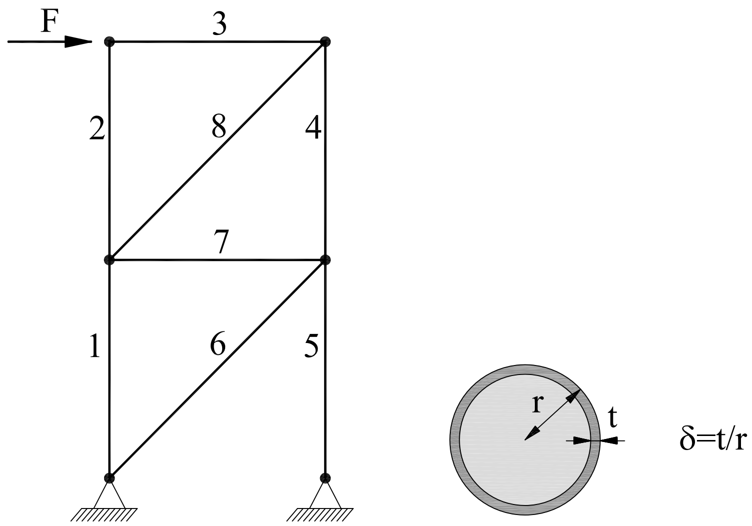

Figure 3.2: Truss system undergoing damage of one member (adapted from Biondini and Restelli, 2008).

Biondini and Restelli (2008) have the same perspective than the one adopted in this report for robustness. In fact when discussing structural robustness, it can be considered any event that may cause damage, from extreme events to just ordinary ones such an environmental exposure causing corrosion. Likewise damage could also be something simple for instance a member failure due to corrosion. The same occurs with structural performances which could be something such as system failure or more trivial such as an increment on the deformations.

The damage considered is the deterioration of section member by corrosion. For the damaged member an external layer of uniform thickness t is removed. Therefore, the amount of damage can be specified by means of the damage index $\delta = t/r \in [0; 1]$.

To assess robustness the authors compared several structural performance indicators on pristine and on damage state. The indicators used were, among others, several stiffness matrix properties, displacements, internal energy measures and pseudo-loads.

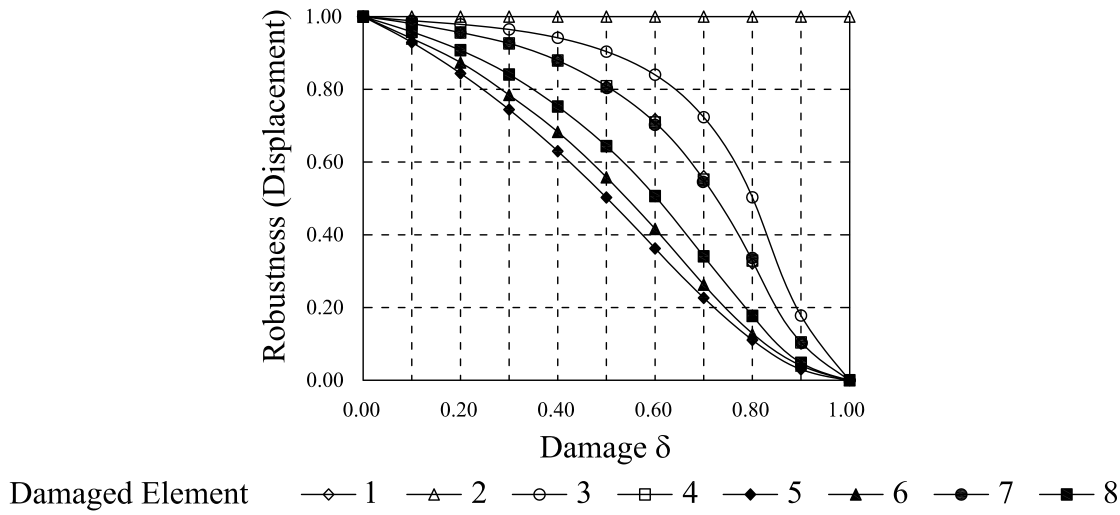

Figure 3.3: Results using displacement and considering the deterioration of all members each at a time (adapted from Biondini and Restelli, 2008).

On Figure 3.3 the results are presented using displacement at the point of force application and considering the deterioration of all members each at a time. As can be seen the structure is full robust if the deterioration on member number 2 is considered. The structure is more susceptible in terms of displacement if member number 5 is corroded.

3.6 Starossek, 2008, 2009

According to Starossek (2008, 2009), progressive collapse resistance can be influenced in various ways. One possibility is trough the structural robustness. In a robust structure, no damage disproportionate to the initial failure will occur.

In terms of probabilities, progressive collapse may be represented as a chain of partial probabilities:

$$P(F) = P(F|D) \times P(D|E) \times P(E) \tag{3.17}$$

where $P(F)$ is the probability of progressive collapse occurrence due to an event E and D represents any kind of damage.

Starossek defines robustness as a property of structure and associated with the term $P(F|D)$. If a structure is robust the probability of failure will not be too much affected by damage occurrence. Robustness here has essentially the same concept adopted in Chapter 2, i.e., robustness concerns about structural response or performance under a damage state.

Having in mind that structural property definition, Starossek also proposes some non probabilistic measures for robustness as follows (Starossek, 2009).

3.6.1 Damage Based Robustness Measure I

The first robustness based measure is given by the expression bellow:

$$R_d = 1 - \frac{p}{p_{lim}} \tag{3.18}$$

where R_d is the damage based measure of robustness, p is the maximum total damage resulting from the assumable initial damage and p_{lim} is the acceptable total damage. A value of one indicates perfect robustness and negative values indicate that the design objectives are not met.

3.6.2 Damage Based Robustness Measure II

The second robustness based measure mainly relates the direct consequences with the initial damage and can be obtained from the follow expression:

$$R_{d,int} = 1 - 2\int_0^1 [d(i) - i]di \tag{3.19}$$

where $R_{d,int}$ is the integral damage-based measure of robustness and $d(i)$ the maximum total damage resulting from and including an initial damage of extent i (dimensionless). A value of one indicates maximum possible robustness and a value of zero indicates total lack of robustness. The curve A represents the more robust structure and curve C the less robust structure (Figure 3.4).

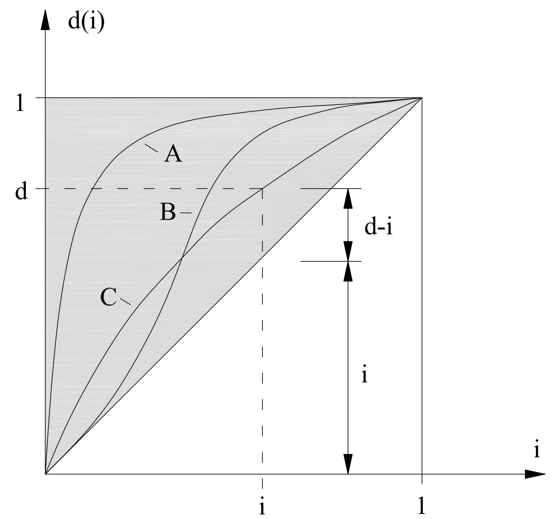

Figure 3.4: Damage Based Robustness Measure II (adapted from Starossek, 2009).

These damage based measures are very expressive, but as was said on Chapter 3, the overall damage is difficult to assess and sometimes is not well correlated with structural performance.

3.6.3 Stiffness Based Robustness Measure

The stiffness based robustness measures proposed stands on a structural property instead of damage and can be computed using the expression bellow:

$$R_s = min_j \frac{detK_j}{detK_0} \tag{3.20}$$

where R_s is the stiffness based measure of robustness, K_j the active system stiffness matrix of structure after removing a structural element or constraint j and K_0 the active system stiffness matrix of the intact structure. Although this measure is based on a structural property it may be not very expressive because it does not correlates always well with the pretended design performances such as load carrying capacity. On the other hand it's easy to calculate.

3.6.4 Energy Based Robustness Measure

More appropriated to impact type progressive collapses, Starossek (2009) proposed an energy based robustness measure given by the follow equation:

$$R_s = 1 - max_j \frac{E_{r,j}}{E_{f,k}} \tag{3.21}$$

where R_e represents the energy based robustness measure, $E_{r,j}$ is the energy released during initial failure of structural element j and contributing to damaging a subsequently affected element k and $E_{f,k}$ is the energy required for failure of subsequently affected element k. A value equal to 1 indicates perfect robustness and negative values indicate failure progression. According with Starossek (2009), usually $E_{r,j}$ is difficult to calculate.

Chapter 4

Proposing a Robustness Index

4.1 Proposed Robustness Indexes

The main objective of this chapter is to analyze the proposed measures for robustness and try to describe the relations between them. Almost all authors referred adopted the structural property perspective, i.e., robustness may be calculated without having in mind structural environment. An exception is Baker robustness index equation 3.14:

$$I_{Rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \tag{3.14}$$

This index depends on structural environmental because both direct and indirect risks depend on exposure and indirect consequences depend on environment.

Although this index is more complete and is able to reflect the all process from exposure to consequences it is very difficult to quantify robustness if concepts such as exposure or consequences, that are also difficult quantify, are employed. A structural engineer would feel much more comfortable if he only would have to deal with structural concepts. This is the major advantage of defining robustness as structural property. On the other hand for the computation of Baker's robustness index, structural response under damage has always to be known.

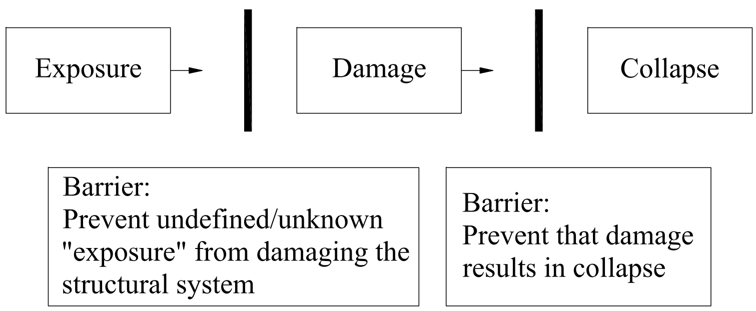

Figure 4.1: Barrier Model (adapted from Sorensen and Christensen, 2006).

To illustrate the above idea one can use the barrier model from Haddon (1980), Ersdal (2005) and Sorensen and Christensen (2006) that represent the structural collapse to a determined exposure (Figure (4.1).

The first barrier protects the structure from environment aggressions and may consist of more general measures including detailed and independent quality control of both design and construction, assessment of loads and design parameters and modeling.

The second barrier deals with the behavior of structural system under damage. The way to enhance this barrier is to design structures robustly and more damage tolerant.

Recalling equation (3.17), it can be said that first barrier acts by reducing the term $P(D|E)$ and for common exposure such as design loads is regulated in standards and codes. The second barrier acts by reducing the term $P(F|D)$ and has not yet been introduced in standards and codes on a quantitative form. The term $P(E)$ for accidental loads and unpredictable exposures it's difficult to manage.

The robustness definition adopted in this report is related only with the second barrier. Instead, Baker's robustness index takes into account with both barriers and goes further by making possible a third barrier between collapse and indirect consequences. This would be possible by introducing a new term, $P(C|F)$, in equations 3.15 and 3.16.

Now let's see how the referred robustness measures relate them selves by using the event tree on Figure 4.2.

Based on the above event tree the probabilistic measures proposed by Frangopol and Curley, Lind and Ghosn and Moses can be rewritten:

- Frangopol and Curley (1987) redundancy index:

$$\beta_R = \frac{\beta_{Intact}}{\beta_{Intact} - \beta_{damaged}} = \frac{\phi(1 - P(F))^{-1}}{\phi(1 - P(F)))^{-1} - \phi(1 - P(F|D))^{-1}} \tag{4.1}$$

- Lind (1995) vulnerability index:

$$V = \frac{P(r_0, S)}{P(r_d, S)} = \frac{1 - P(F)}{1 - P(F|D)} \tag{4.2}$$

- Ghosn and Moses (1998) target system values:

  If damage, D, represents member failure, Ghosn and Moses (1998) target system may be rewritten in follows forms,

$$\Delta\beta_{ult} = \beta_{ult} - \beta_{member} = \phi(1 - P(F))^{-1} - \phi(1 - P(D|E)P(E))^{-1} \geq 0.85 \tag{4.3}$$

  and

$$\Delta\beta_{damaged} = \beta_{damaged} - \beta_{member} = \phi(1 - P(F|D))^{-1} - \phi(1 - P(D|E)P(E))^{-1} \geq -2.70 \tag{4.4}$$

  If failure, F, represents excessive deformation instead, the above expression can be suggested:

$$\Delta\beta_{serv} = \beta_{serv} - \beta_{member} = \phi(1 - P(F))^{-1} - \phi(1 - P(D|E)P(E))^{-1} \geq 0.25 \tag{4.5}$$

From the above expressions the follow conclusion can be extracted. The indexes proposed by Frangopol and Curley (1987) and Lind are almost the same, represent a

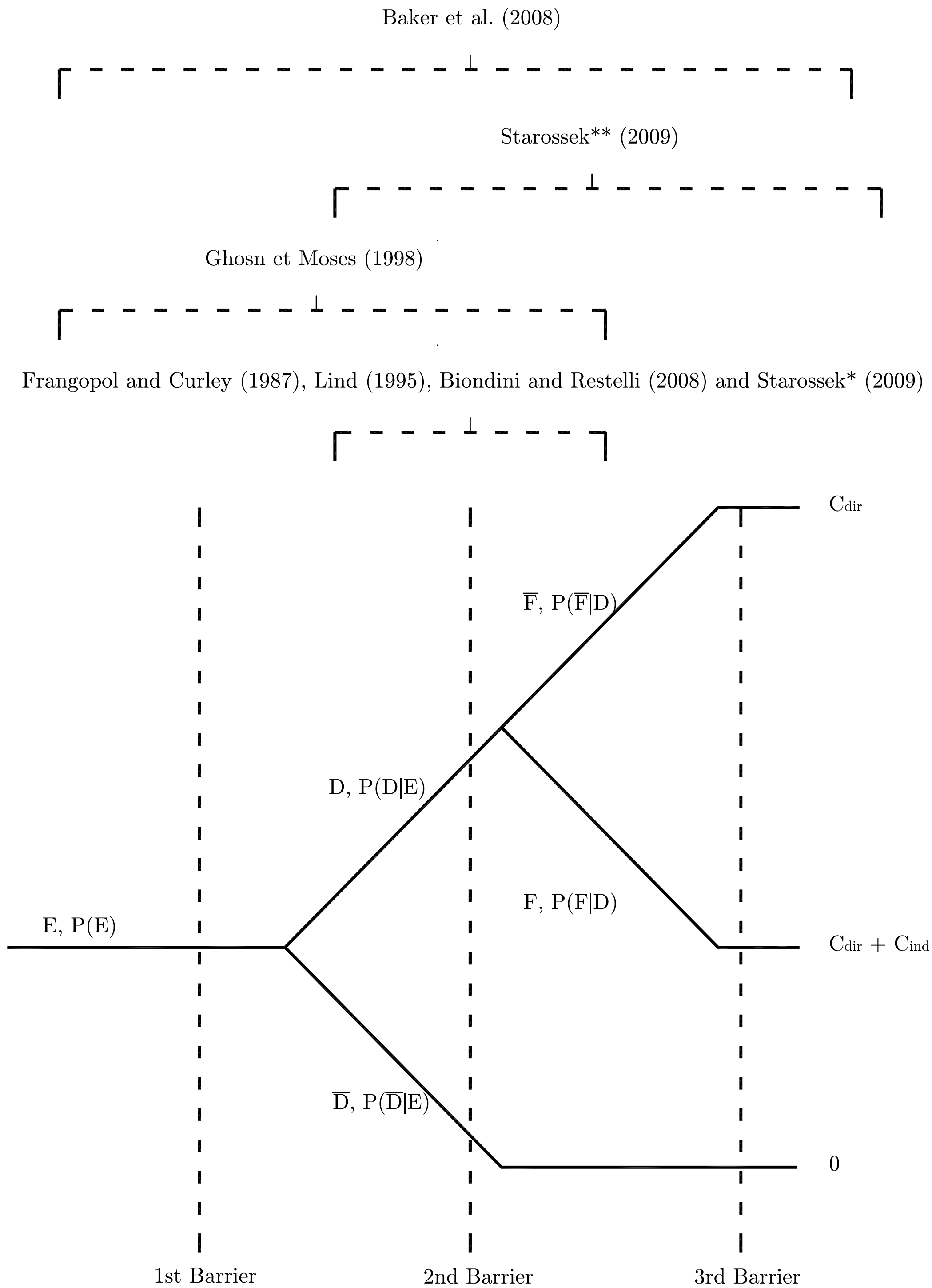

Figure 4.2: Event Tree (* Stiffness based measure; ** Damaged based measures).

relation between the failure probability of the intact and damage system and refer only to the second barrier of the event tree.

Ghosn and Moses (1998) proposal go further by introducing the therms $P(E)$ and $P(D|E)$ related with member and reflecting the first barrier. Ghosn and Moses (1998) also propose values for the target limits although just for highway bridges.

Baker et al. (2008) index also accounts with both direct and indirect consequences resulting from damage and collapse and for that it is a risk measure.

Biondini and Restelli (2008) analyze the structural behavior when system is subjected to a continuous damage scenario. The authors attempt to describe the relation between structural performance and damage. So, if F represents a performance indicator of structural behavior and D the damage, the referred approach can be synthesized on the follow form:

$$\rho = \frac{F(D = d)}{F(D = 0)} \tag{4.6}$$

where ρ is the robustness indicator that depends on the structural performance indicator studied and on the level of damage.

This framework reveals of great interest when damage is a continuous variable as occurs in many real situation such as damage resulting from deterioration or excessive deformation, among others. The extension to a probabilistic measure is also possible if damage could be considered as a random variable. The probabilistic measure should result on the above expression:

$$\rho = \frac{P(F|D = d)}{P(F|D = 0))} \tag{4.7}$$

which is the inverse of the Vulnerability index as defined by Lind.

Regarding Starossek (2009) measures, it can be said that the first two (damage based robustness measure I and II) mainly measure the direct consequences C_{dir} resulting from an initial damage D. Although they are deterministic, these measures can by very useful when computing Baker et al. (2008) robustness index.

The stiffness based robustness measure proposed by Starossek (2009) reflects the changes on structural behavior when damage occurs and for that is closer to Biondini and Restelli (2008) proposal. The energy based robustness measure is more appropriate to predict a progressive collapse but it is difficult to generalize.

In conclusion it can be said that, from the analyzed measures for robustness, Frangopol and Curley (1987); Lind (1995); Biondini and Restelli (2008); Starossek (2009) (only the stiffness based measure) measures are more concerned with structural performance. The other ones attempt to quantify also the consequences resulting from the losses on structural performance resulting from an initial damage. When combined with probabilities these are risk measures.

4.2 Proposing a Robustness Index

So robustness as defined here in this report pretend to describe behavior of structures after damage occurs. If concern is about a specific structural function F and if a specific damage D is considered, a useful form to measure robustness would be a robustness index

$\beta_R(F, D)$ defined as:

$$\beta_R(F, D) = \phi \left[1 - P(F|D)\right]^{-1} \tag{4.8}$$

This index could be calibrated for different performances F and damages D depending on structural type and exposure. Alternatively one may want a unique robustness index independent of damage or performance. In that case the follow expression may be suggested:

$$\beta_R = \phi \left[1 - \int_x \int_y f_{F|D}(y|x) dy dx)\right]^{-1} \tag{4.9}$$

In this case F and D would represent the overall design performance functions and damage scenarios respectively.

Regard that equation 4.9 is very similar to equation 3.16 without the terms C_{Ind}, $f_{D|EX_{BD}}$ and $f_{EX_{BD}}$.

Although Starossek (2009) and Biondini and Restelli (2008) do not present probabilistic measures the concept would be easily extended as presented by Frangopol and Curley (1987) with the deterministic redundancy factor R and the probabilistic redundancy factor β_R.

The main idea of a deterministic approach would be to compare specific performance indicators F with the structure intact ($D = 0$) and damaged ($D = d$):

$$R(f, D) = R\left[F(D = 0), F(D = d)\right] \tag{4.10}$$

where R would be a robustness factor. The form of R would be defined depending on the performance in study.

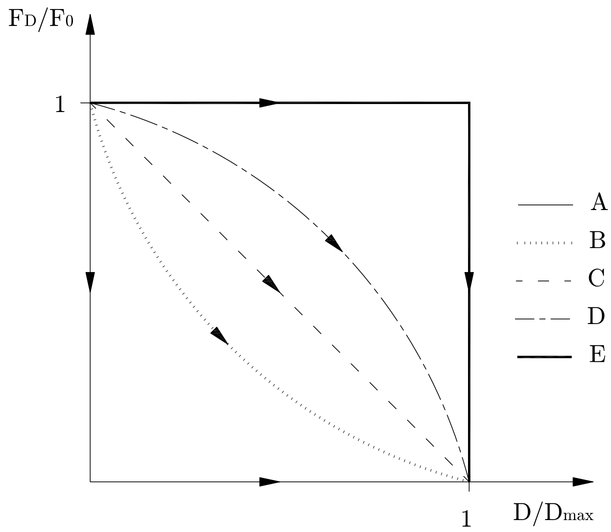

Figure 4.3: Normalized structural response as a function of normalized damaged.

When damage D is a continuous variable it would be preferable to analyze the degree of performance lost across the overall dominium of D. In this case the follow index may be suggested:

$$R = \int_0^1 f_d(x) dx \tag{4.11}$$

where f_d is the normalized response of the structure obtained by the ratio F_D/F_0 as a function of the normalized damage variable obtained by the ratio D/D_{max}.

In this case the robustness index would vary from zero to one with correspondence with extreme cases A and E respectively. For curve A, a minimum damage would lead to total performance lost, and for curve E only the maximum damage possible would cause some difference on structural response. The curve C would represent reference robustness of 1/2.

On Figure 4.3 in all cases represented the maximum damage leads to total lost on structural performance. This methodology is also valid in other situations where the maximum damage does not correspond to total lost in performance or in situations where the collapse happens before maximum damage occurs.

Chapter 5

Corroded reinforced concrete structures - Methodology

5.1 Introduction

The main objective of this chapter is to present an example of robustness assessment on a typical structure. As was discussed in the previous chapter, the definition adopted considers robustness as a property of the structure. Robustness measures the variance on structural performance as a function of damage. This can be done on a probabilistic or deterministic framework.

The damaged scenario considered on this chapter will be deterioration of reinforced concrete structures due to corrosion. Structural response to ultimate limit states such as load carrying capacity will be studied.

To represent adequately effects of corrosion on reinforced concrete structures it is necessary to take account with some undesirable mechanisms such as net area reduction of reinforcement and expansion around reinforcements due to corrosion products accumulation. This last phenomenon leads to damage, cracking and splitting of the surrounding concrete and degradation of steel-concrete bond responsible for stress transfer between both materials.

To perform such a study, an advanced finite elements methodology will be used coupled with advanced constitutive models for materials. First an analysis of the corroded cross section will be carried out, subsequently the results obtained will be used to enrich a 2D model of the structure with the properties of the deteriorated section.

The competence of the referred methods to reproduce the behavior of corroded reinforced concrete was demonstrated by comparing obtained numerical results with results obtained experimentally (Sánchez et al. (2008)).

The following sections are not intended to be an exhaustive and complete explanation of the models and methodologies used. A brief explanation of the tools used will be presented in order to show how they can be used to simulate the behavior of corroded reinforced concrete. Full details can be found in Sánchez et al. (2008) and Oliver et al. (2008).

5.2 Finite element methodology

On this section the advanced finite element methodology used to simulate corrosion phenomenon will be briefly explained. This methodology was introduced by Sánchez et al. (2008) and Oliver et al. (2008), where a full and detailed explanation can be found.

The methodology employed considers a two-step analysis. In the first step a finite element analysis of the structure cross section is carried out. In this analysis the formation and accumulation of corrosion products are simulated by an expansion of steel bars (Figure 5.1). Steel bars are modeled using a linear elastic constitutive relation and they are coupled to concrete through an interface model that regulates the shear stress transference between the two materials. For concrete, an isotropic continuum damage model was used enriched with kinematics provided by the strong discontinuities theory. The combination of these two approaches, for modeling concrete behavior, permits the development of cracking caused by corrosion as a result of concrete deterioration (Figure 5.1 (c)). The results obtained during the cross section analysis are than used to build a 2D structural model.

To model reinforced concrete a continuum strong discontinuity approach coupled with mixture theory as described in Oliver et al. (2008) was used. The mixture theory consists on modeling reinforced concrete as a composite material, constituted by a plain concrete matrix reinforced with embedded long fiber bundles which represent the steel bars. Matrix failure is modeled on the basis of the continuum damage model enriched with the results provided by the cross section analysis. For the steel bars an elasto-plastic with slipping-fiber option model was used.

The degradation of bond between concrete and steel bars, as a result of corrosion, plays an important role on load carrying capacity. To account for that, an empiric model of bond degradation presented by Bhargava et al. (2007) was used to complement the slipping-fiber model used on the structural analysis. At the same time the reduction of

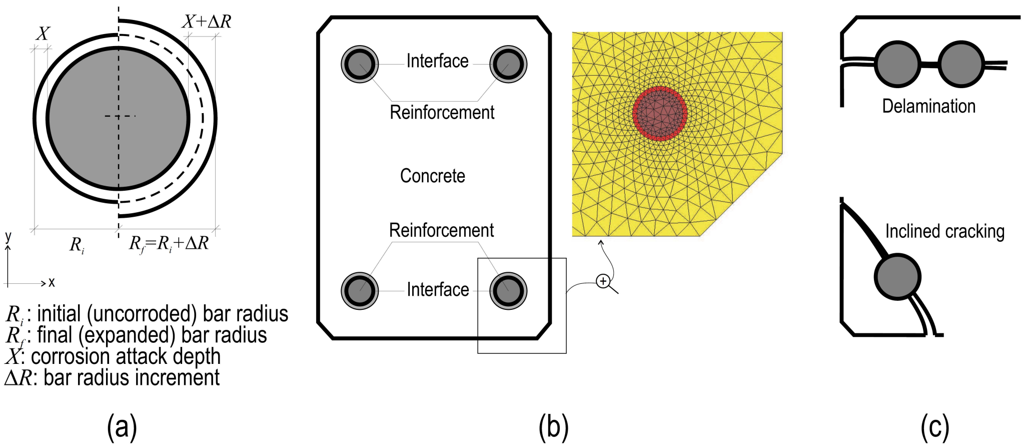

Figure 5.1: Plane strain 2D model: (a) Corrosion expansion mechanism. (b) Numerical model idealization. (c) Typical pattern of cracks. (adapted from Sánchez et al., 2008)

reinforcement area was taken in account as a function of corrosion depth X.

On Figure 5.2, a fluxogram represents schematically the methodology used.

On the next sections a more detailed explanation of the models features will be presented

5.3 CSDA - Continuum Strong Discontinuity Approach (Oliver et al., 2002; Oliver and Huespe, 2004)

As earlier described, steel expansion, resulting from corrosion products accumulation, produces cracking and splitting on surrounding concrete. Concrete cracks are, from a macroscopical point of view, discontinuities that can be characterized as jumps on the displacement field across material, denoted also as strong discontinuities.

In the context of traditional Continuum Mechanics dealing with discontinuities is a hard task, so since the ultimate load carrying capacity is largely influenced by the formation and pattern of cracking, an advanced methodology with competences do deal with cracking effectively had to be chosen.

The continuum strong discontinuity approach ($CSDA$) is an advanced and recent methodology which is equipped with recent and advanced ingredients in order to make it an efficient and robust strategy for solving complex three-dimensional multi crack problems.

According with $CSDA$ methodology if a body H experiences a strong discontinuity (*i.e.*, a crack formation) across the surface S described by the normal n, the displacement field will experience a jump (Figure 5.3). If the surface S divides the solid in two domains Ω^+ and Ω^- then the displacement $u(x)$ and the compatible strain field $\epsilon(x)$ may be written in the form:

$$u(x) = \overbrace{\bar{u}(x)}^{continuous} + \overbrace{H_S(x)[\![u]\!](x)}^{discontinuous} \; ; \; H_S(x) = \begin{cases} 1 & \forall\, x \in \Omega^+ \\ 0 & \forall\, x \in \Omega^- \end{cases} \tag{5.1}$$

$$\epsilon(x) = \nabla^{sym} u(x) = \underbrace{\bar{\epsilon}(x)}_{regular} + \underbrace{\delta_S(x)\left([\![u]\!] \otimes n\right)^{sym}}_{singular} \tag{5.2}$$

where $u(x)$ is a continuous function, $[\![u]\!](x)$ represents the displacement jump across the discontinuity S and $H_S(x)$ is the step function. The strain field shows a singular term, the second one in equation (2), given by the Dirac's delta distribution $\delta_S(x)$.

The above strategy, to include strong discontinuities, refers only to the kinematics aspect of the problem but it is possible to couple it with an effective constitutive relation to adequately represent concrete behavior.

5.4 Isotropic Continuum Damage Model (Oliver et al., 1990)

Concrete exhibits a complex constitutive behavior especially in the neighborhoods of cracking. Continuum Damage Models have been widely accepted as an alternative to

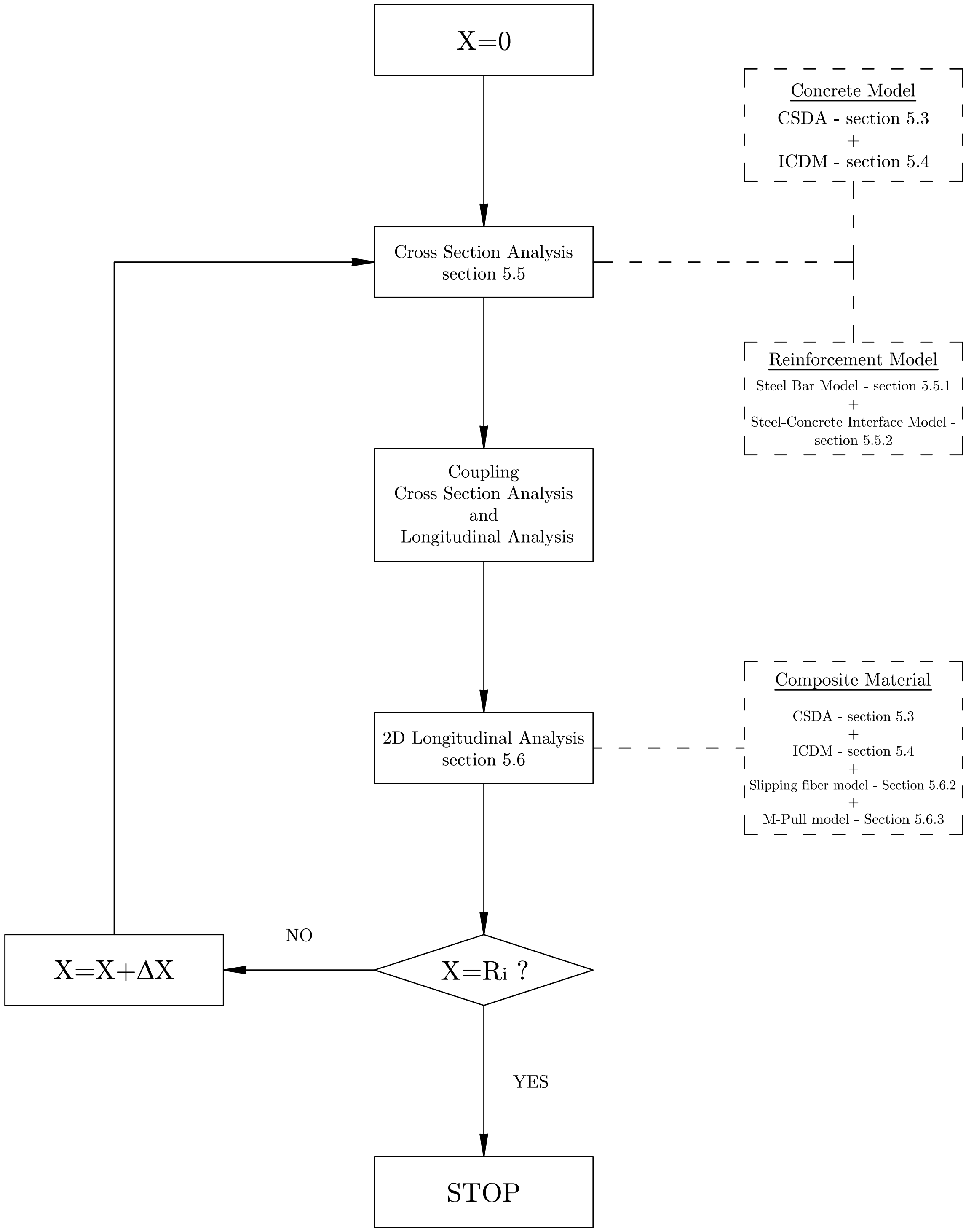

Figure 5.2: Corrosion analysis methodology fluxogram.

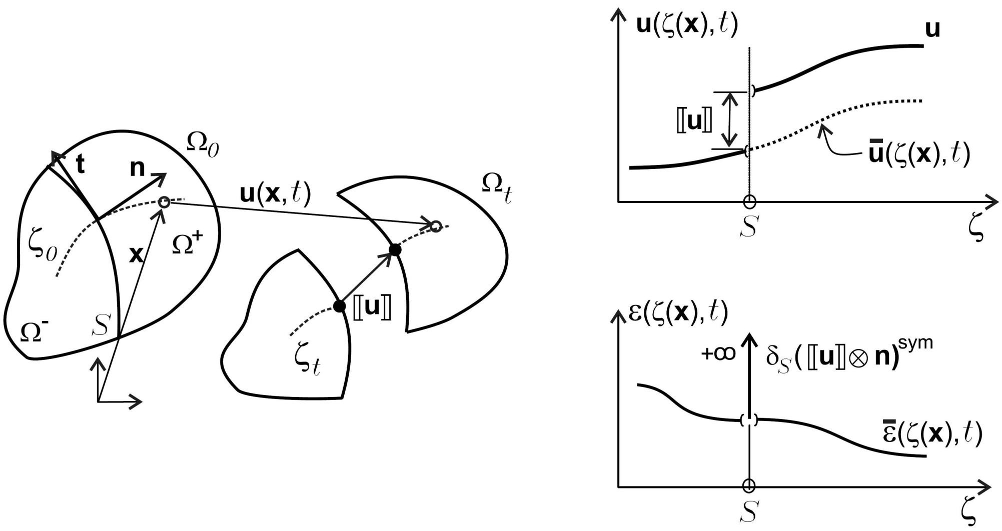

Figure 5.3: CSDA kinematics. Jump on displacement field (adapted from Oliver et al., 2006).

deal with this complex behavior.

On of the main concepts of continuum damage mechanics is consideration that physically the degradation of material properties is the result of the initiation, growth and coalescence of micro cracks or micro voids. One may model this process by introducing an internal damage or deterioration variable d, which can be a scalar or a tensorial quantity.

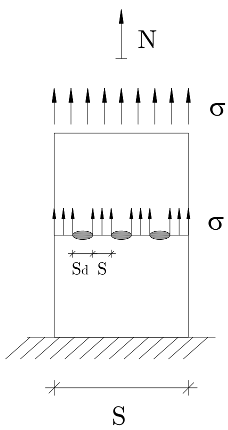

Figure 5.4: Equilibrium of a damaged section.

The equilibrium of the damaged section on Figure 5.4 may be written on the follow form:

$$N = \sigma \cdot S = \bar{\sigma} \cdot \bar{S} \tag{5.3}$$

where N is the tension force, σ is the homogenized apparent stress, S is the total nominal area surface, $\bar{S}$ is the effective surface and $\bar{\sigma}$ the effective stress. Based on the above expression one may write:

$$\sigma = \frac{\bar{S}}{S} \cdot \bar{\sigma} = \frac{S - S_d}{S} \cdot \bar{\sigma} = \left(1 - \frac{S_d}{S}\right) \cdot \bar{\sigma} = (1 - d) \cdot \bar{\sigma} \tag{5.4}$$

where S_d is the damaged area. On a one-dimensional problem d physically represents the ratio of damage surface over total surface area at a local material point (equation 5.5) and $(1 - d)$ is a reduction factor that relates effective with homogenized and apparent stress.

$$d = \frac{S_d}{S} \tag{5.5}$$

As the effective stress is given by:

$$\bar{\sigma} = E \cdot \epsilon \tag{5.6}$$

where E is concrete Young's modulus and ϵ is the strain, equation 5.4 can be rewritten on the follow form:

$$\sigma = (1 - d) \cdot E \cdot \epsilon = E_d \cdot \epsilon \tag{5.7}$$

When d takes zero value it corresponds to undamaged state. When d is equal to one it means completely damaged stage is reached. When d is a scalar variable, it represents an isotropic damage case, i.e., the mechanical behavior of micro cracks is independent of their orientation.

Another aspect of interest of the isotropic continuum damage model is that damage surface S_d cannot diminish, which implies that:

$$\dot{S}_d \geq 0 \Rightarrow \dot{d} \geq 0 \tag{5.8}$$

where $\dot{S}_d$ and $\dot{d}$ represent temporal derivates of both S_d and d.

Another important concept of the damage model is that degradation is initiated when the strain ϵ (or stress σ) exceeds the initial damage threshold ϵ_0 (σ_0):

$$d = 0 \quad \text{if} \quad \left\{ \begin{array}{c} \epsilon \leq \epsilon_0 \\ or \\ \sigma \leq \sigma_0 \end{array} \right. \tag{5.9}$$

Graphically the model can be represented in Figure 5.5 for a uniaxial tension case.

Summarizing the isotropic continuum damage model for three-dimensional case:

$$\underline{\underline{\sigma}} = (1 - d)\underline{\underline{\underline{\underline{C}}}} : \epsilon \tag{5.10}$$

$$0 \leq d \leq 1 \ ; \ \dot{d} \geq 0 \tag{5.11}$$

where $\underline{\underline{\sigma}}$ represents second order stress tensor and $\underline{\underline{\underline{\underline{C}}}}$ represents the fourth order isotropic elastic tensor.

Now it is important to analyze how damage variable d evolves after degradation is initiated when the strain or stress reach a specific threshold. An efficient form to do so is defining d as a function of an internal variable r as follows:

$$d(r) = 1 - \frac{q(r)}{r} \quad ; \quad 0 \leq d(r) \leq 1 \tag{5.12}$$

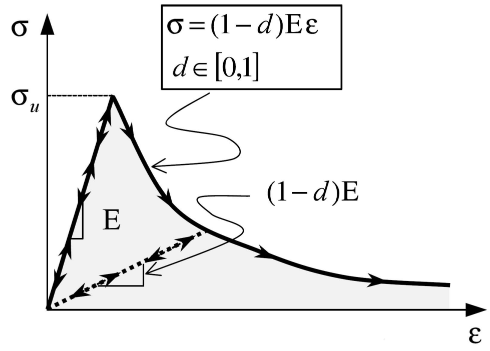

Figure 5.5: Isotropic continuum damage model (adapted from Oliver et al., 2002).

where $q(r)$ is a hardening variable that determines how d evolutes and helps defining the damage criterion, i.e., the elastic domain, which could be defined as:

$$\tau\left(\underline{\underline{\sigma}}\right) - q(r) \leq 0 \tag{5.13}$$

where $\tau\left(\underline{\underline{\sigma}}\right)$ is a stress norm defined as:

$$\tau\left(\underline{\underline{\sigma}}\right) = \left\|\underline{\underline{\sigma}}\right\| \tag{5.14}$$

The equation 5.13 is the damage criterion and defines the elastic domain. On the three-dimensional case, it represents a damage surface.

The internal variable r variable may be defined as follows:

$$r \in [r_0, \infty] \ ; \ r_{|t=0} = r_0 \ ; \ \dot{r} \geq 0$$

when $r = r_0$ corresponds to undamaged state and $r = \infty$ corresponds to full damaged state. The time positive derivate of r means that damage cannot decrease.

The hardening variable $q(r)$ defines how material behavior evolves when damage occurs. Several possibilities may be considered as can be seen on Figure 5.6. H is the hardening parameter defined by $dq(r)/dr$. When H is positive the material is hardening, when H is negative the material is softening. Similarly, when H is positive the surface defined by damage criterion is rising and vice-versa.

5.5 Cross Section Analysis

As earlier describe, in both cross section and 2D longitudinal structural analysis, concrete was modeled by means of an isotropic continuum damaged model combined with a strong discontinuity approach. On the cross section analysis steel rebars and interface elements were modeled as described in the following paragraphs.

5.5.1 Steel bar model

During the deterioration analysis of the corroded cross section a standard linear elastic constitutive relation was assumed for the steel bars. The corrosion products accumulation

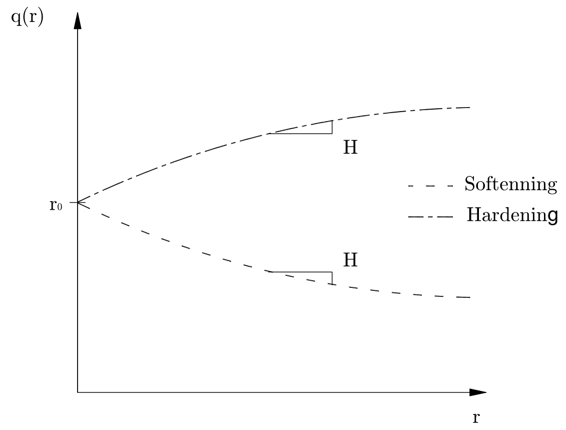

Figure 5.6: Material hardening and softening.

and expansion effect was simulated by a volumetric deformation of the steel bars ϵ_0. Assuming a plain strain state, the total strains ϵ can be obtained by the sum of the strains due to stresses ϵ^e and due to the referred volumetric expansion ϵ^0:

$$\epsilon = \nabla^{sym} u(x) = \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \\ \epsilon_{zz} \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \\ 0 \end{bmatrix} = \overbrace{\begin{bmatrix} \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz}) \\ \frac{1}{E}(\sigma_{yy} - \nu\sigma_{zz} - \nu\sigma_{xx}) \\ 2\frac{1+\nu}{E}\sigma_{xy} \\ \frac{1}{E}(\sigma_{zz} - \nu\sigma_{xx} - \nu\sigma_{yy}) \end{bmatrix}}^{\epsilon^e} + \overbrace{\begin{bmatrix} \mathcal{D} \\ \mathcal{D} \\ 0 \\ 0 \end{bmatrix}}^{\epsilon^0} \tag{5.15}$$

The variable $\mathcal{D}$ represent the dilational component, depends on the corrosion attack depth X and can be obtained from the follow expression:

$$\mathcal{D} = \frac{R_f^2 - R_i^2}{2R_i^2} \tag{5.16}$$

where R_i is the initial bar radius and R_f is the final bar radius. Assuming incompressibility of the accumulated corrosion products and taking the bar radius increment equal to corrosion depth, the final bar radius can be computed as:

$$R_f = R_i + \Delta R \tag{5.17}$$

Figure 5.1 (c) illustrate the process.

During the cross section analysis and for a determined corrosion depth X, the dilation $\mathcal{D}$ is applied incrementally during the nt time steps required to perform the non-linear analysis.

5.5.2 Steel-Concrete interface model

To simulate the mechanism of shear stress transference between steel and concrete the interface model represented on Figure 5.7 was used.

This type of element coupled with CSDA methodology allows the expected separation between steel and concrete materials when high levels of corrosion are reached. As

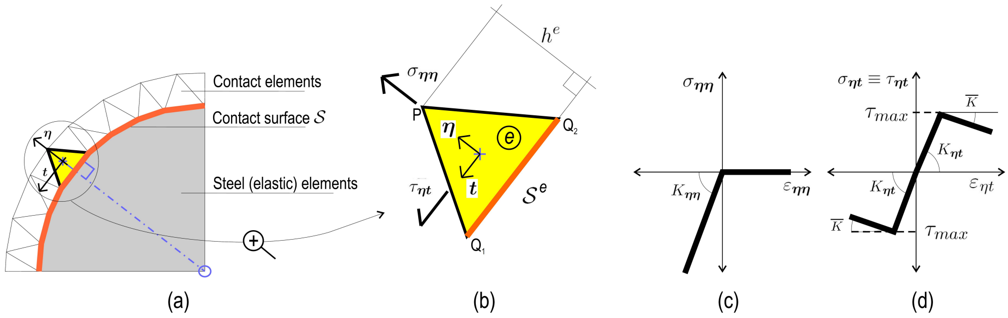

Figure 5.7: Steel-Concrete interface model adopted from Sánchez et al. (2008)

can be seen on Figure 5.7 the interface model is characterized by four parameters: the normal stiffness $K_{\eta\eta}$, the maximum adherence stress τ_{max}, the shear stiffness K_{η} and the hardening/softening shear modulus $\bar{K}$.

5.6 2D Longitudinal Analysis

As earlier stated, in the 2D longitudinal analysis, reinforced concrete was modeled by means of a composite material constituted by a matrix and long fibers which represent the steel bars (Figure 5.8). A more detailed explanation of the model will be presented in the next subsections.

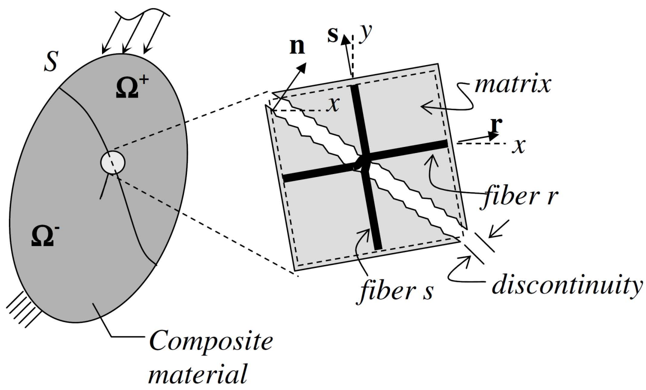

Figure 5.8: Composite material model (adapted from Oliver et al., 2008).

5.6.1 Composite material model

On the mixture theory, reinforced concrete is assumed to be a composite material constituted by a matrix, which represent the concrete, and long fibers which represent the steel rebars. According to the basic hypothesis of the mixture theory , a composite material is a continuum in which each infinitesimal volume is occupied simultaneously by all constituents behaving as a parallel mechanical system. As a consequence, all the constituents are subjected to the same composite strain ϵ and the stresses are given by the weighted sum, in terms of the volume fraction, of the stresses of each constituent. In Figure 5.9 an illustration of the composite material components is presented. Although the model can accounts with dowel action effect, in the present study this option was not considered.

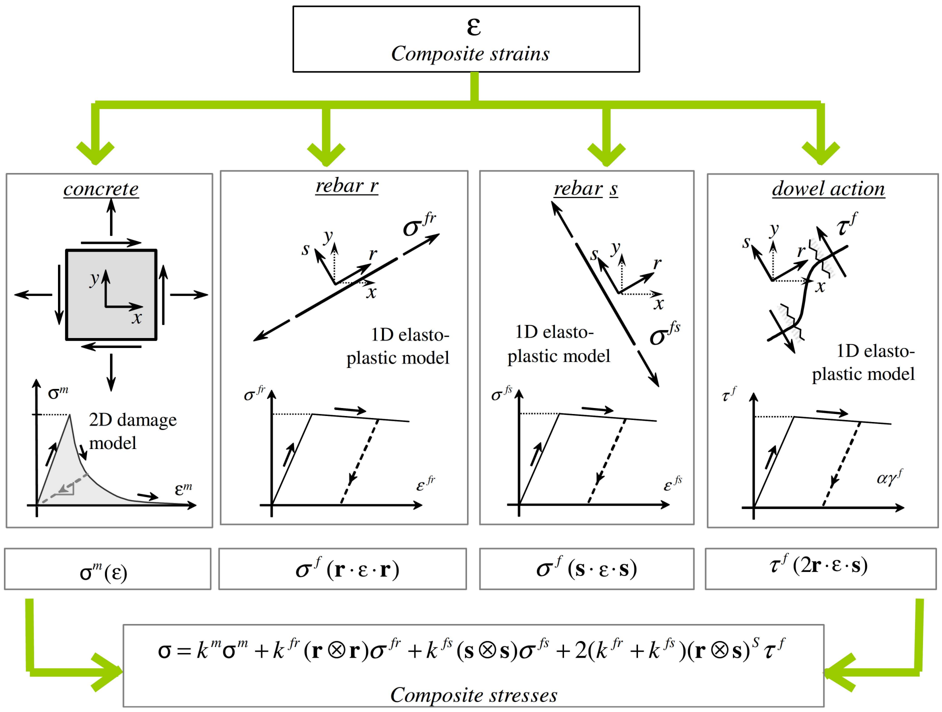

Figure 5.9: Composite material model components (adapted from Oliver et al., 2008).

For the constitutive model of the concrete matrix, the isotropic continuum damage model already described was adopted. The rebar bond-slip effects were taken into account via a combination of the uniaxial elasto-plastic model, for the rebars, and a uniaxial slip dissipative model for the interface concrete-rebars, resulting in a unique constitutive model called *slipping-fiber*.

5.6.2 Slipping-fiber model

To account for the bond-slip effect, it is assumed that slipping-fiber ϵ^f strain is composed of two parts:

$$\epsilon^f = \epsilon^d + \epsilon^i \tag{5.18}$$

where ϵ^d is the fiber mechanical deformation and ϵ^i is the deformation due to interface sliding.

Assuming a two-component serial system constituted by the fiber and the interface, as shown in Figure 5.10, the corresponding slipping-fiber stress σ^f is identical to the stress of each component:

$$\sigma^f = \sigma^d = \sigma^i \tag{5.19}$$

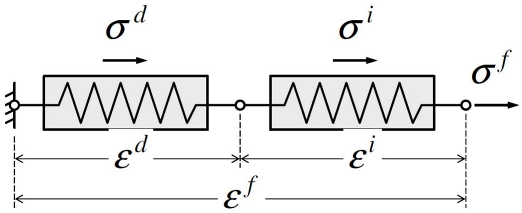

Figure 5.10: Slipping-fiber model (adapted from Oliver et al., 2008).

where σ^d is the fiber stress and σ^i is the interface stress. On both cases the stress-strain relation can be obtained via an one-dimensional elasto-plastic model hardening/softening. The resulting constitutive behavior for the slipping-fiber is also an elasto-plastic model with the following characteristics (Figure 5.11):

$$\sigma_y^f = min(\sigma_y^d, \sigma_{adh}^f) \tag{5.20}$$

$$E^f = \frac{1}{\frac{1}{E^d} + \frac{1}{E^i}} \tag{5.21}$$

$$H^f = \begin{cases} H^d & if \;\; \sigma_y^d < \sigma_{adh}^f \\ H^i & if \;\; otherwise \end{cases} \tag{5.22}$$

in which E^d and σ_y^d are the steel Young's modulus and yield stress, respectively, E^i is the interface elastic modulus and σ_{adh}^f is the interface bond limit stress. In this study, the hardening/softening model option characterized by parameters H^f,H^d,H^i respectively for slipping-fiber, fiber and interface was not considered.

Regard that when $E^i \to \infty$ and $\sigma_y^d < \sigma_{adh}^f$, the system provides only the mechanical behavior of the fiber, reproducing a perfect combination between concrete and reinforcement bars.

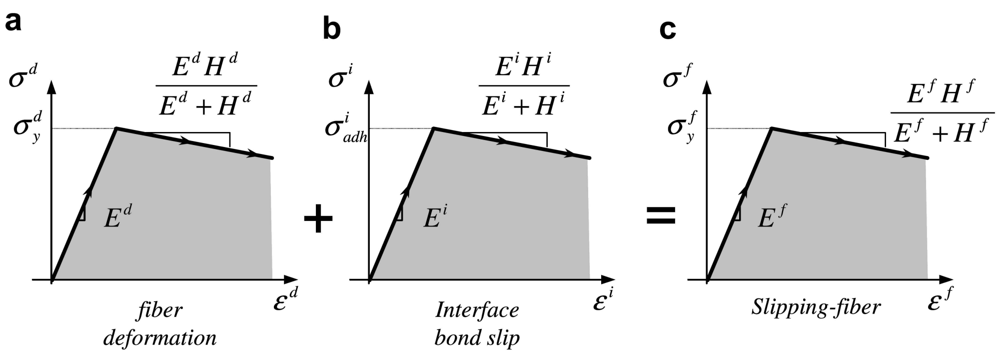

Figure 5.11: Slipping-fiber model composition (adapted from Oliver et al., 2008).

The parameters required to characterize the slipping-fiber model can be obtained from pullout tests. In the present study perfect adhesion between steel bars and concrete was considered and a rigid-plastic behavior for the interface was adopted just for the uncorroded state.

5.6.3 Bond-Strength deterioration (Bhargava et al., 2007)

Corrosion plays a fundamental role on bond strength weakening. Literature research (Al-Sulaimani et al., 1990; Cabrera, 1996; Rodriguez et al., 1994; Almusallam et al., 1996; Amleh and Mirza, 1999; Auyeung et al., 2000; Lee et al., 2002; Fang et al., 2004) showed that research, on the influence of corrosion on bond strength, has used a wide variety of bond specimens and bar types, resulting in the wide range reported bond strengths for the same levels of corrosion. Therefore, choosing a model for bond deterioration resulted is an hard task.

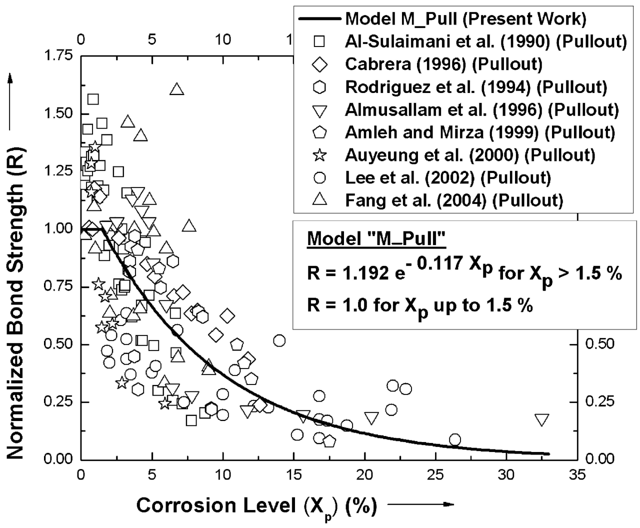

Figure 5.12: Normalized Bond strength as a function of corrosion level for experimental data of pullout tests (adapted from Bhargava et al., 2007).

To improve this situation an empirical model, developed by Bhargava et al. (2007) and based on a large set of different pullout experimental tests, was adopted. On Figure 5.12 it can be seen all the experimental data considered and the *M-Pull* model adopted.

M-pull model gives the normalized bond strength R as a function of corrosion level X_p. Corrosion level X_p is the loss of weight of reinforcing bar expressed as a percentage of original rebar weight. M-Pull model can be resumed on equation 5.23.

$$R(X_p) = \begin{cases} 1.0 & if \;\; X_p \leq 1.5\% \\ 1.192 \cdot e^{-0.117 X_p} & if \;\; X_p > 1.5\% \end{cases} \tag{5.23}$$

Notice that experimental presented data on Figure 5.12 do not account with stirrups effect on bond strength deterioration. Although the studied examples had stirrups the presented model was used. This option was taken due to lack of consistent information about the subject.

Chapter 6

Robustness of Corroded Reinforced Concrete Structures - Numerical Example

6.1 Introduction

The main objective of this chapter is to present two practical examples of real reinforced concrete structures subjected to corrosion. The effects of corrosion will be simulated using the methodology presented in the previous chapter. Finally, using the robustness deterministic measure proposed on Chapter 4, the two proposed structural solutions will be compared. For this propose, damage considered will be the corrosion level X_p and the function performance studied will be the load carrying capacity.

6.2 Design Solution

Having in mind the potential size of the numerical models resulting from modeling real reinforced concrete structures using the methodology presented, two small foot bridges were adopted for the present study.

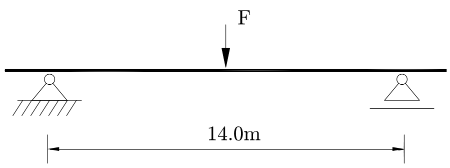

Figure 6.1: 2D Structural model.

The same structural model was chosen for the two design solutions for a direct comparison between them. The structural model considered was a simply supported beam with 14.0m of free span subjected to a midspan concentrated load as represented on Figure 6.1.

The requisites when designing the two different solutions were a load carrying capacity as required in CEB (1993) and a walking path of 2.0m of width. Based on this, two cross sections were designed. The first one is a slab and the second one is a I beam, both represented on Figure 6.2.

As can be seen on Figure 6.2, the I beam is a much more slender solution. Although if the longitudinal models are compared, the slab solution has a slenderness of $L/h = 14.0/0.70 = 20$ and the I beam solution has a slenderness of $L/h = 14.0/1.80 = 7.8$.

The effect of shear was neglected on the present study and the transverse reinforcement was over designed. The deterioration of this type type of reinforcement was also neglected.

6.3 Cross Section Analysis

On this section cross section analysis will be described. On a first step, both cross sections were discretized in finite elements. Smaller finite elements were assigned to the regions close to steel rebars. Due to the size of smaller finite elements, about 2mm, only a zoom of a rebar zone can be presented on Figure 6.3.

The same material properties were used for the two design solution. The mechanical properties adopted for concrete are resumed in Table 6.1.

Table 6.1: Concrete Material Properties

Material	f_{ctm} (MPa)	f_{cm} (MPa)	E (GPa)	ν	G_f (N/m)
Concrete	3.0	30.0	30	0.20	0.10

During cross section analysis, constant corrosion depth X for all steel rebars was considered. This means that different values of dilatation parameter $\mathcal{D}$ were assumed for

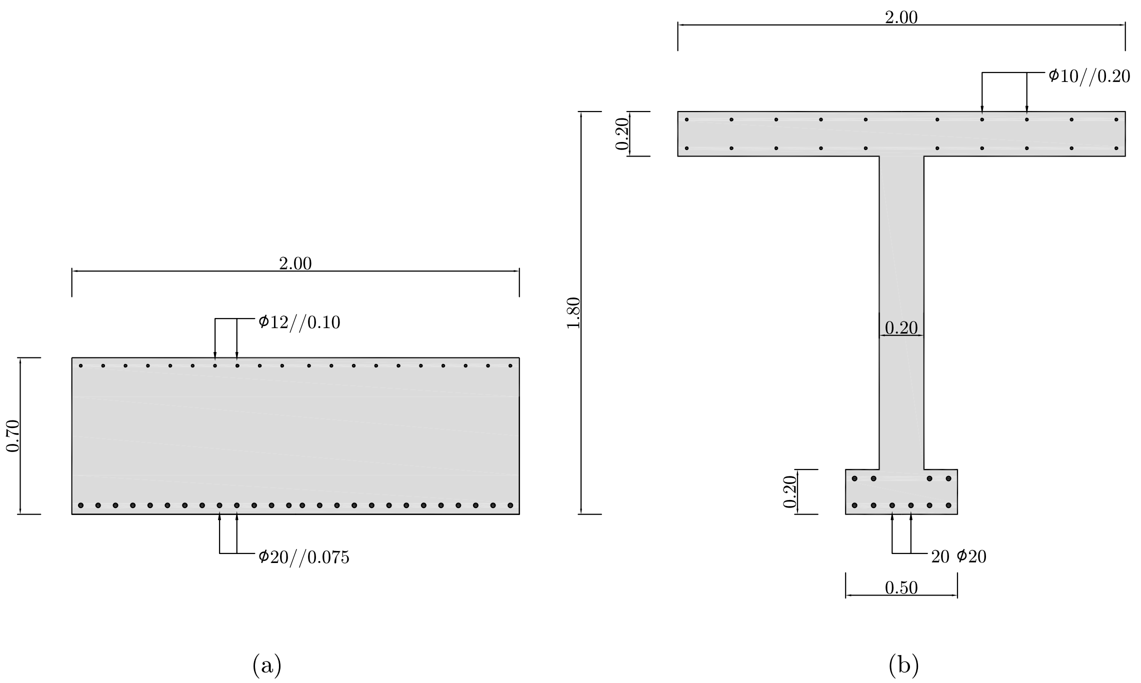

Figure 6.2: Cross Sections. (a) Slab solution; (b) I beam solution.

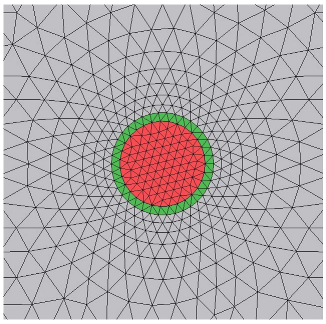

Figure 6.3: Finite elements discretization around steel rebars.

each different steel bar radius. Similarly for each analysis step, corrosion levels X_p were higher for small radius rebars.

As the analysis started and the corrosion levels increase concrete around steel rebars became damaged. As was presented on Chapter 5, damaged concrete begins loosing strength. When variable damage is equal to 1 cracks start appearing. On Figure 6.4 damage concrete pattern slab solution for half section is shown. The damage around steel bars is evident.

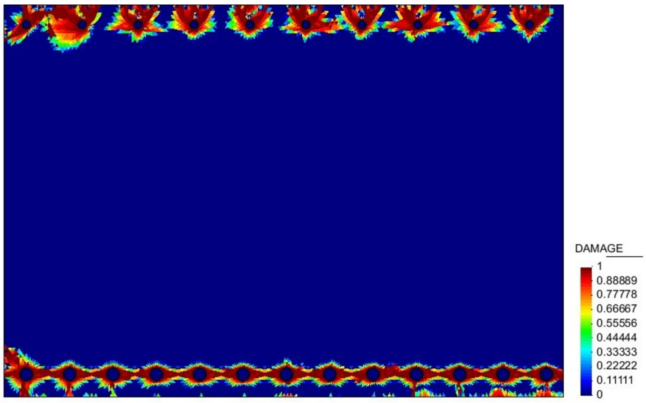

Figure 6.4: Concrete damaging due to corrosion for slab design solution.

On Figure 6.5 isodisplacement lines are presented for the same step of the cross section analysis presented on Figure 6.4. When isodisplacement lines tend to concentrate it means that a crack appeared on that zone. By analyzing Figure 6.5, it is possible to observe, on the bottom of the slab, the delamination of concrete cover, due to a crack formation between all rebars. On the cross section top this happened only on the most left side bars. On the other bars the cracks just crossed the concrete cover.

By comparing Figures 6.5 and 6.4 it is also possible to conclude that isodisplacement lines concentrate on most damaged areas. When a crack crosses all the section it means that the part of concrete that remains outside the crack is no longer monolithic with the rest of the section. On this study, the influence of these parts of concrete, on the cross section resistance, was neglected as the influence of stirrups on crack developing. This assumption was taken into account during the 2D longitudinal analysis by considering these parts of concrete with damage equal to one. As was seen, damage equal to one means that concrete has completely lost all its resistance.

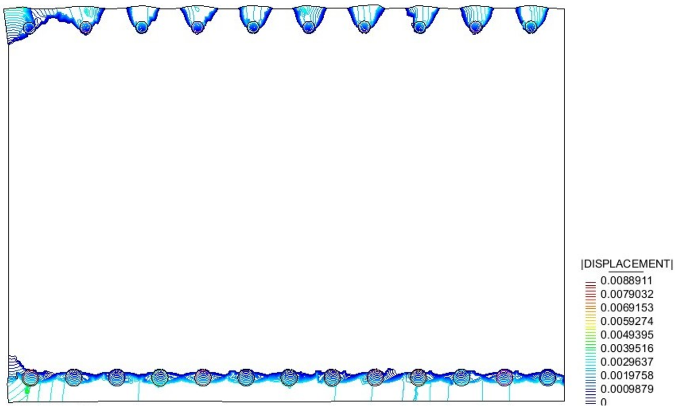

Figure 6.5: Isodisplacement lines for slab design solution.

For the I beam cross section design solution, damage pattern and isodisplacement lines are presented on Figure 6.6 for an advanced corrosion attack level.

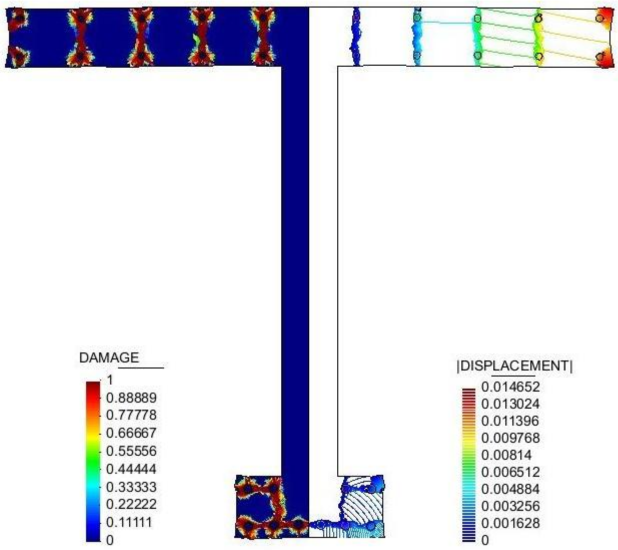

Figure 6.6: Damage and isodisplacement lines for I beam design solution.

Figure 6.6 shows that corrosion produces several cracks. On the top flange, several cracks appeared crossing all its depth. On the bottom flange, delamination of concrete cover occurs on both levels of steel bars. Another crack appeared connecting these two levels of reinforcement. In this case, the loss of monolithicism is much more important because section inertia and effective reinforcement area will be drastically reduced. The effect of stirrups was also neglected on this case due to the increased complexity involved.

6.4 2D longitudinal model for structural analysis

Using the strategy presented in Chapter 5, longitudinal models for structural analysis were built. For concrete material the same properties adopted on cross section analysis were considered. To characterize the slipping-fiber model, during the non corroded stage, an elastoplastic constitutive behavior was considered for steel bars with a Young modulus E^d equal to $200GPa$ and yielding stress σ^d equal to $400MPa$. For the interface a rigid-plastic model with a yielding stress also equal to $400MPa$ was adopted. These assumptions result in perfect adhesion in the non corroded stages.

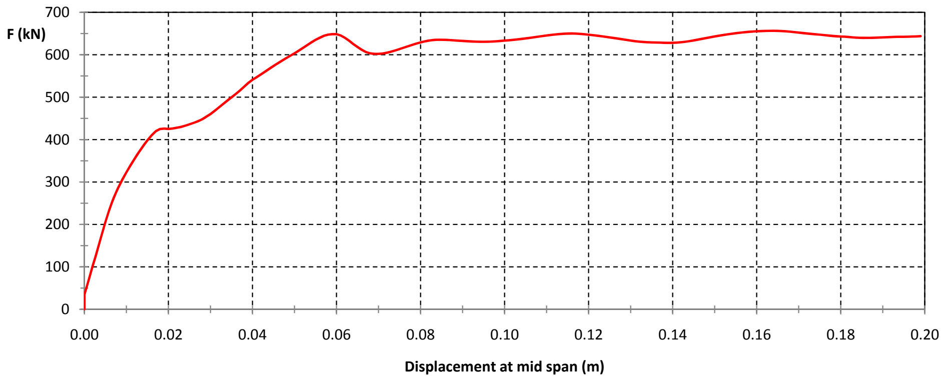

Figure 6.7: Uncorroded stage force - displacement diagram for slab design solution.

To compare structural behaviors of corroded and uncorroded states, for each design solution, a first run of the longitudinal model was performed. On Figure 6.7 is presented the force-displacement diagram for slab solution.

Three behavior stages can be observed. The elastic behavior on first stage correspond to maximum load of about 450kN. The second stage correspond to crack spreading and goes to a maximum load of about 650kN. On this stage it is possible to observe the loss on structural rigidity due to cracking. On the third stage, structure can no longer sustain load increment as the second stage ends with the yielding of steel on the bottom bars. Thus the third stage correspond to mechanism development that consist on a plastic hinge formation under the load application point, where flexural moments are higher.

On Figure 6.8 (a) and (b), results of damage and horizontal isodisplacement lines, corresponding to the last step of the analysis, are presented. In this case, there are damaged zones where isodisplacement lines do not concentrate, so it looks like that there are no crack formation on that damaged zones. In fact on the last step of the analysis the concentration of isodisplacement lines is much higher on the midspan where the larger cracks are. Thus it becomes difficult to observe smaller cracks on other damage areas and Figure 6.8 (b) only show two big cracks around the beam mid span.

On Figure 6.8 (c) it is shown an horizontal displacement diagram of the slab bottom surface. The diagram is aligned with isodisplacement lines on Figure 6.8 (b) so it is possible to observe the jump on the displacement field due to crack formation where isodisplacement lines concentrate. This jump occurrence was characterized on Chapter 5 as a strong discontinuity.

For the I-beam cross section solution the same results are now presented on Figures 6.9 and 6.10. In this case it is also possible to observe the damage, in the cross section web concrete, due to shear effect.

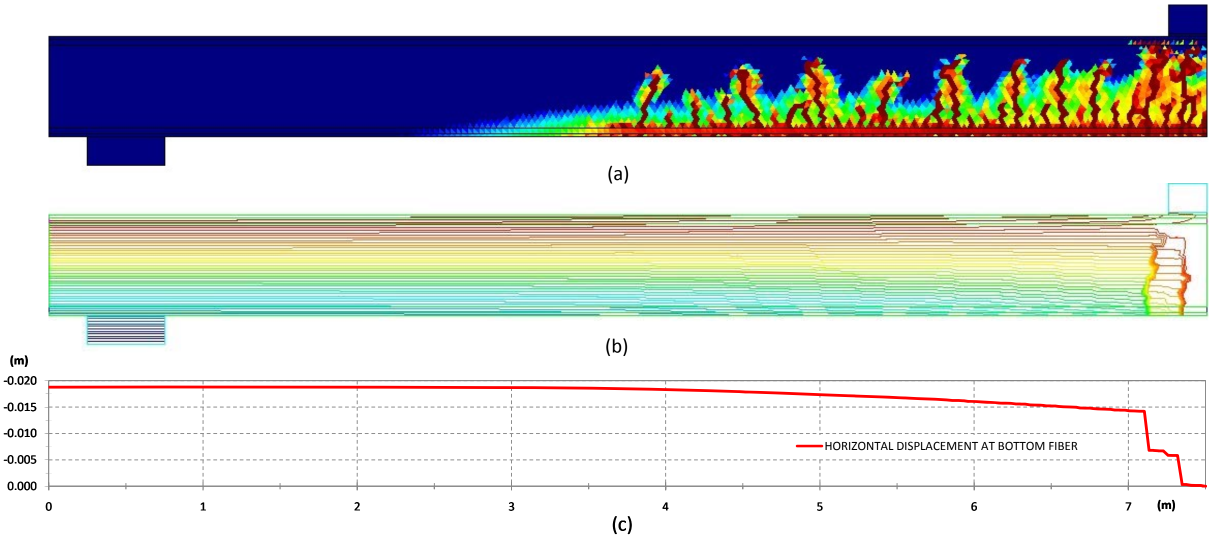

Figure 6.8: Slab design solution results for the uncorroded stage. (a) Damage on Concrete. (b) Horizontal (x-direction) isodisplacemet lines. (c) horizontal displacement at cross section bottom fiber points.

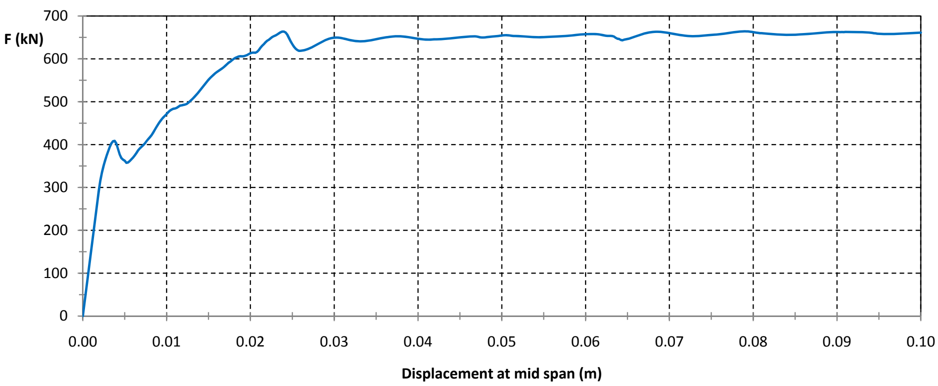

Figure 6.9: Uncorroded stage force - displacement diagram for I-beam design solution.

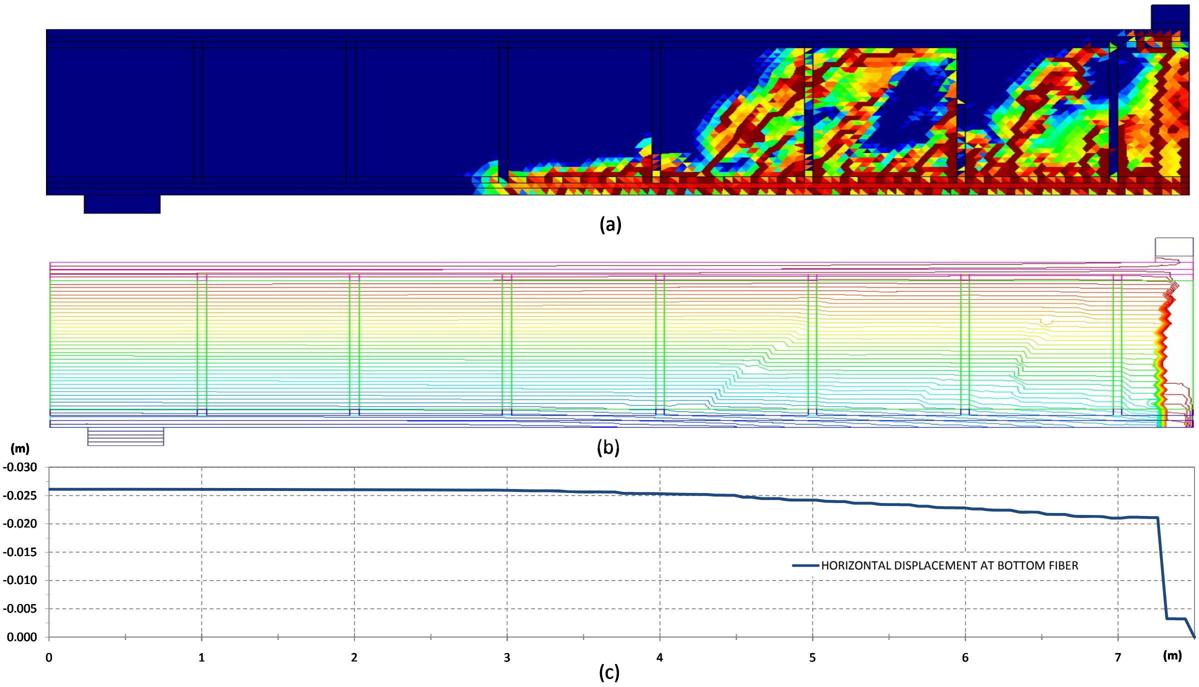

Figure 6.10: I-beam design solution results for the uncorroded stage. (a) Damage on Concrete. (b) Horizontal (x-direction) isodisplacemet lines. (c) horizontal displacement at cross section bottom fiber points.

6.5 Coupling cross section and 2D structural analysis

On this section the coupling strategy between cross section and 2D structural analysis will be explained.

Cross section analysis is performed until advanced corrosion depths X are reached. In fact after the formation of the crack pattern presented on previously in section 6.2, there is no need to go further with the analysis because no more cracks will appear. Just crack's width will increase without additional significant damage for concrete.

For each attack depth X and for the correspondent step of the cross section analysis the results were examined. If there were cracks crossing all over the section than the part of section that remains outside the crack was considered with damage equal to 1. At the same time if there were steel bars on these parts of the section, they were not taken into account for the longitudinal analysis.

After this task is completed, the cross section was divided into horizontal slices and the average value of damage d variable, for each slice, was computed. The average damage was then projected on the 2D longitudinal model.

On Figure 6.11 the all process with respect to the coupling strategy is resumed.

For each corrosion depth X, when the longitudinal structural analysis started the concrete was already damaged with values picked from cross section analysis. Before run this second analysis, it was necessary to compute the corrosion level X_p and the effective reinforcement area for each steel bar. It was also necessary to compute the loss on bond strength to characterize the slipping-fiber model. After these tasks are completed, all

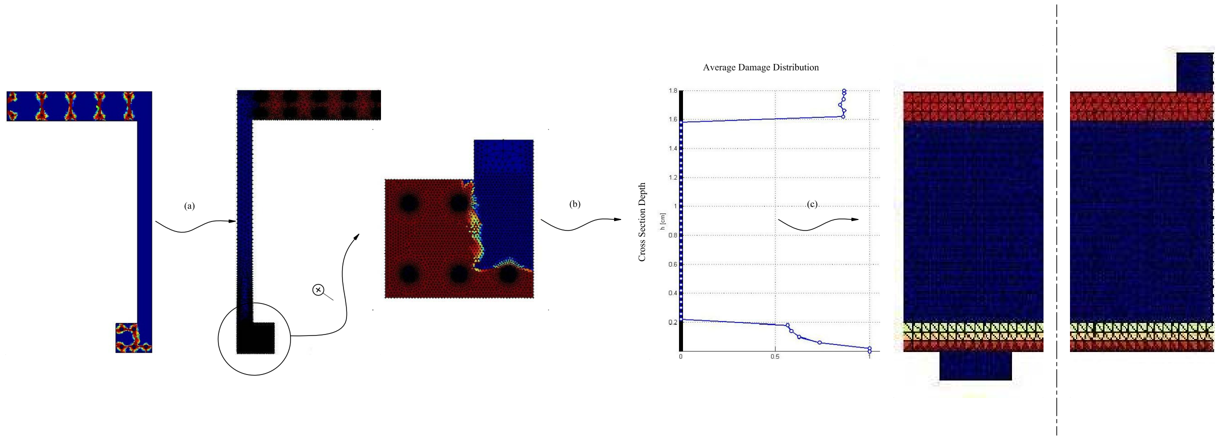

Figure 6.11: Coupling Strategy. (a) Defining regions with $d = 1$. (b) computing Average damage distribution along cross section depth. (c) Coupling the cross section damage results with the 2D longitudinal model.

features of the longitudinal model were updated and the analysis could than start.With this methodology it was possible to account with the follow effects produced by steel corrosion:

1. expansion and accumulation of corrosion products resulting on concrete deterioration by means of damage variable d;

2. concrete cracking and lost of monolithicism of section;

3. bond strength deterioration;

4. reinforcement effective area reduction.

6.6 Results

On Figure 6.12, load-displacement diagrams are presented for several corrosion levels X_p for the slab design solution. As the corrosion level increases, the second behavior stage, which correspond to crack spreading, tend to become shorter. For a corrosion depth of $X_p = 7.92\%$ the second stage does not exist and after the first crack appears steel bars immediately reach the yielding stress. In fact, when the first crack occurs, stresses pass from concrete to steel bars. As steel bars are significantly corroded and the adherence to concrete is deteriorated, rebars cannot support the demanded stresses, and the load falls to lower levels compatible with the deteriorated state.

So, for advanced corrosion states, steel rebars play a small role on ultimate load of the beam. They increase slightly the maximum load on the uncraked stage and they are responsible for post–peak carrying load capacity. For a corrosion level of 100% only concrete contributes for the cross section resistance and than there is no post peak carrying capacity.

On Figure 6.13 same results are presented for the I-beam design solution. The conclusions on structural behavior are basically the same. Although in this case it is possible

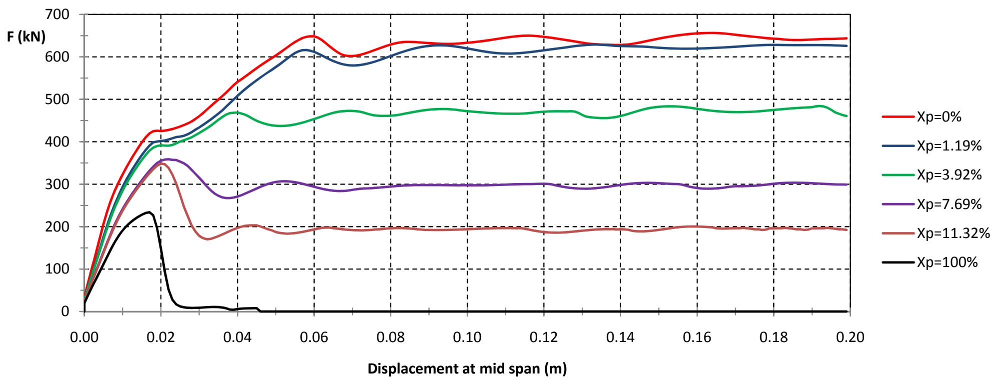

Figure 6.12: Force - displacement diagram for slab design solution and for several corrosion levels X_p.

to observe a bigger difference between corroded and uncorroded states load carrying capacities. In this case the peak load decreases from about $650kN$ to $150kN$ respectively for $X_p = 0\%$ and $X_p = 100\%$. On the slab design case the load reduction was from about $650kN$ to $225kN$ for the same corrosion levels. This means that, without steel bars, the slender cross section in I shape is weaker, or by other words, that the influence of steel bars on the overal resistance is higher on the I-beam case.

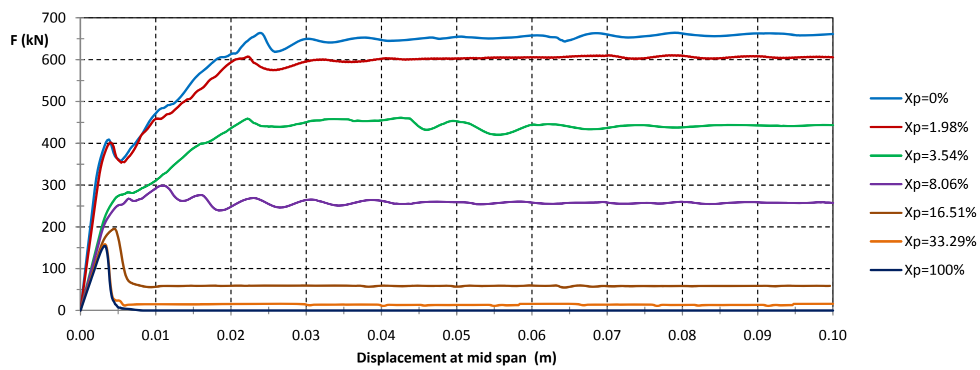

Figure 6.13: Force - displacement diagram for I-beam design solution and for several corrosion levels X_p.

Now let's apply the deterministic robustness measure proposed on Chapter 4 in order to compare the susceptibility to corrosion of both design solutions.

As was previously defined, robustness is a property of the structure that measure the degree of loss on structural performance due to damage. In this case structural performance will be the peak load and damage will be the corrosion level X which measures the loss on effective reinforcement area.

On Figure 6.14 the normalized peak load carrying capacity F/F_{max} of the structures is plotted against corrosion level X_p. The normalized bond strength of the slipping-fiber model was also plotted on the same graphic and against corrosion level X_p.

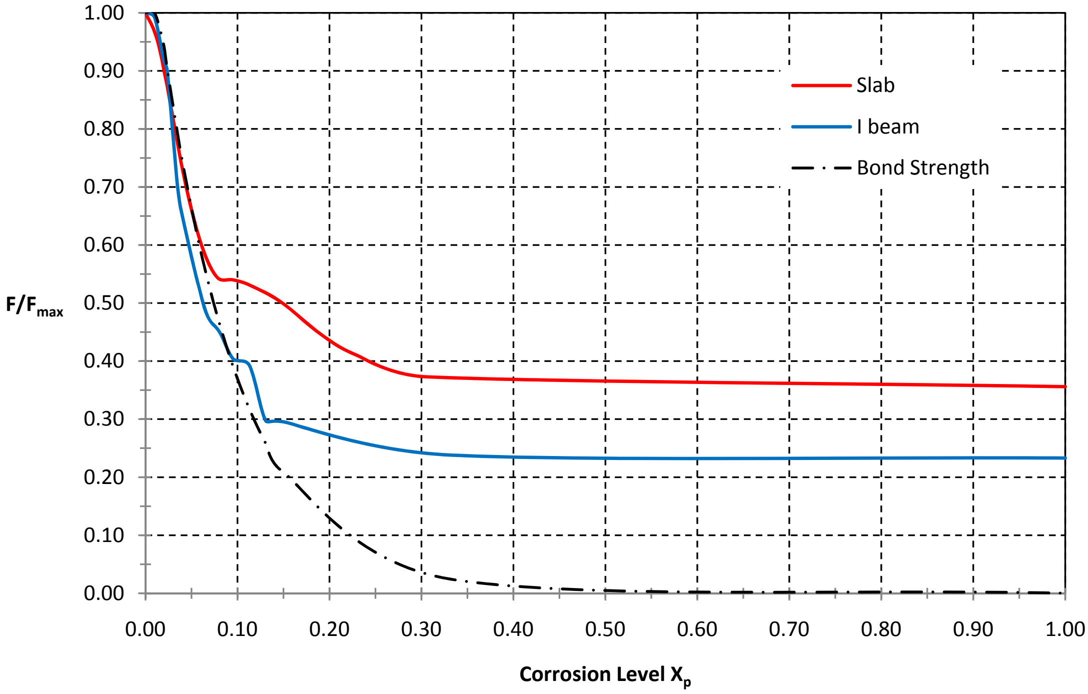

Figure 6.14: Normalized peak load carrying capacity F/F_{max} and bond strength $\frac{(\sigma_y^i)^{corroded}}{(\sigma_y^i)^{uncorroded}}$ as a function of the corrosion level X_p.

From the analysis of Figure 6.14 it is possible to conclude that bond strength plays a major role for the first corrosion states. For $X_p \leq 0.075$ and $X_p \leq 0.15$ for slab and I-beam designs respectively, bond strength reduction domains above all other phenomenons causing peak load decreasing.

Another conclusion taken from Figure 6.14 is the fact that the I-beam curve is more irregular than the slab curve. The small jumps on curves slope observed are due to cross section cracks occurrence with loss of monolithicism as a result of expansion and accumulation of corrosion products. For the slab, the bottom and upper concrete cover spalling occurrence contrasts with the detaching of large parts of both bottom and upper flanges on the I-beam case.

For corrosion levels higher than $X_p = 0.075$ and $X_p = 0.15$ for slab and I-beam designs, respectively, both curve slopes tend to decrease because bond strength degradation ratio also tends to decrease and reinforcement starts losing influence on the overall resistance of the cross section. For $X_p = 0.40$ bond strength is almost null and for both cases concrete is the only material providing resistance to cross section. As was said, for I-beam design, steel reinforcement has a major influence on the resistance of the cross section, so the loss on load carrying capacity is higher in this case. When steel reinforcement is totally corroded, $X_p = 1$, the load carrying capacity is 36% and 23% of the load carrying capacity on the uncorroded state, respectively for slab and I-beam designs.

The robustness of both solution can be assessed using equation 4.11:

1. For the slab design:

$$R = \int_0^1 F/F_{max}(X_p)dX_p = 0.42 \tag{6.1}$$

2. For the I-beam design:

$$R = \int_0^1 F/F_{max}(X_p)dX_p = 0.29 \tag{6.2}$$

As was expected robustness result higher for the slab design case. To confirm the compe-

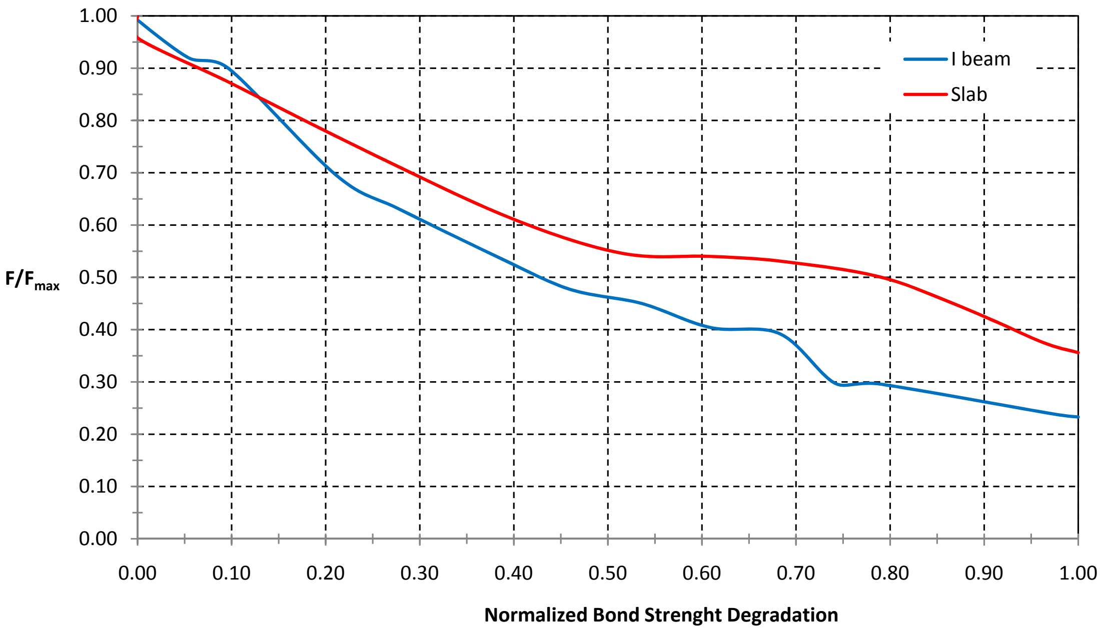

Figure 6.15: Normalized peak load carrying capacity F/F_{max} as a function of normalized bond strength deterioration $D_{\sigma_y^i}$.

tence of the above measure another damage variable to assess structural robustness was considered. As was observed bond strength degradation plays an important role on load carrying capacity. On Figure 6.15 normalized load carrying capacity F/F_{max} is plotted against normalized bond strength degradation $D_{\sigma_y^i}$ given by:

$$D_{\sigma_y^i} = 1 - \frac{(\sigma_y^i)^{corroded}}{(\sigma_y^i)^{uncorroded}} \tag{6.3}$$

where $(\sigma_y^i)^{uncorroded}$ $(\sigma_y^i)^{corroded}$ represent the bond strength of the slipping-fiber model respectively on corroded and uncorroded states.

The loss on structural performance (load carrying capacity) has a linear relation with damage (bond strength degradation) for both cases. Applying on more time equation 4.11:

1. to the slab design
$$R = \int_0^1 F/F_{max}(D_{\sigma_y^i})dD_{\sigma_y^i} = 0.61 \tag{6.4}$$
2. and to the I-beam design
$$R = \int_0^1 F/F_{max}(D_{\sigma_y^i})dD_{\sigma_y^i} = 0.51 \tag{6.5}$$

the slab design result again more robust.

If an unreinforced structure was considered on the present study, robustness would be equal to 1 because corrosion would not affect the load carrying capacity. This is coherent with the adopted definition, i.e., a full robust structure is one that does not loose any performance under damage.

Chapter 7

Conclusions and Future Work

At the present time, robustness is not well defined and much controversy still remains around the subject.

On this report several proposed robustness definitions were analyzed and discussed. It was concluded that some proposals define robustness as a structural property and others define it as a property of both structure and environment. From the analysis carried out, based on a event tree and on the barrier model, it was also concluded that all authors proposals are closely related. The authors, that consider robustness as a property of the structure, are mainly concerned about structural performance after damage occurrence. Other authors regard also for structural susceptibility to an event and for damage direct and indirect consequences.

In this report, the theory of robustness being a structural property was adopted and the follow definition was proposed:
Robustness is a measure of the degree of structural function lost after a damage occurs. Function can be of any kind, from service limit states to ultimate limit states. Damage can vary from a simple degradation state to a more serious damage as a column or a beam failure.
In order to assess robustness as defined above, a probabilistic index was proposed on equations 4.8 and 4.9. A deterministic measure was also defined by means of equation 4.11.

To illustrate the suggested definition for robustness and methods for quantifying it, an example of a corroded reinforced concrete structure was presented. The example consisted of two simply supported beams with different cross sections. For the first beam a slab design solution was adopted, and for the second one an I-beam design solution was considered.

The structural performance under analysis was the load carrying capacity and the damage considered was the corrosion level of the bottom reinforcement.

To capture effects of reinforcement corrosion, an advanced finite element methodology was performed. The finite element analysis were based on three main features:

1. an isotropic continuum damage model in order to represent concrete behavior;
2. a continuum strong discontinuity approach to consider concrete cracking;
3. and the mixture theory for the composite material, concrete and reinforcement, simulation.

With the purpose of capturing concrete damage and cracking occurrence due to corrosion products accumulation, an in-plane analysis of the cross section was firstly performed. The results of the corroded cross section were than coupled with a longitudinal model of the simply supported beams and the structural analysis were finally performed for several corrosion levels. The effect of bond strength deterioration was also considered by means of the *M-pull* model.

Cross section analysis revealed that the reinforcement corrosion produces spalling of concrete cover on the slab design solution. On the I-beam design case, corrosion produces cracks crossing all over both bottom and upper flanges weakening more significantly the section resistance.

Structural longitudinal analysis showed that bond strength deterioration is the major cause of load carrying capacity on both beams.

For the two design solutions normalized load carrying capacity was plotted as a function of corrosion level and as a function of bond strength deterioration. On both cases, the robustness assessment, using the deterministic measure proposed on equation 4.11, demonstrated that slab solution is more robust. This is mainly due to lost of I-beam cross section integrity due to cracking of bottom and upper flange. The other reason is the important role played by the steel reinforcement on flexural resistance of I-beam cross section. Consequently reinforcement corrosion has a higher impact on the load carrying capacity of the I-beam.

As future developments it would be of great interest to perform a reliability analysis and assess the robustness of the presented examples with the probabilistic measure proposed on equation (4.8). It would be also of great interest to illustrate the proposed definition and measures by means of more examples, i.e., considering more damage scenarios and different structural performance indicators.

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